Programming on Encrypted Data

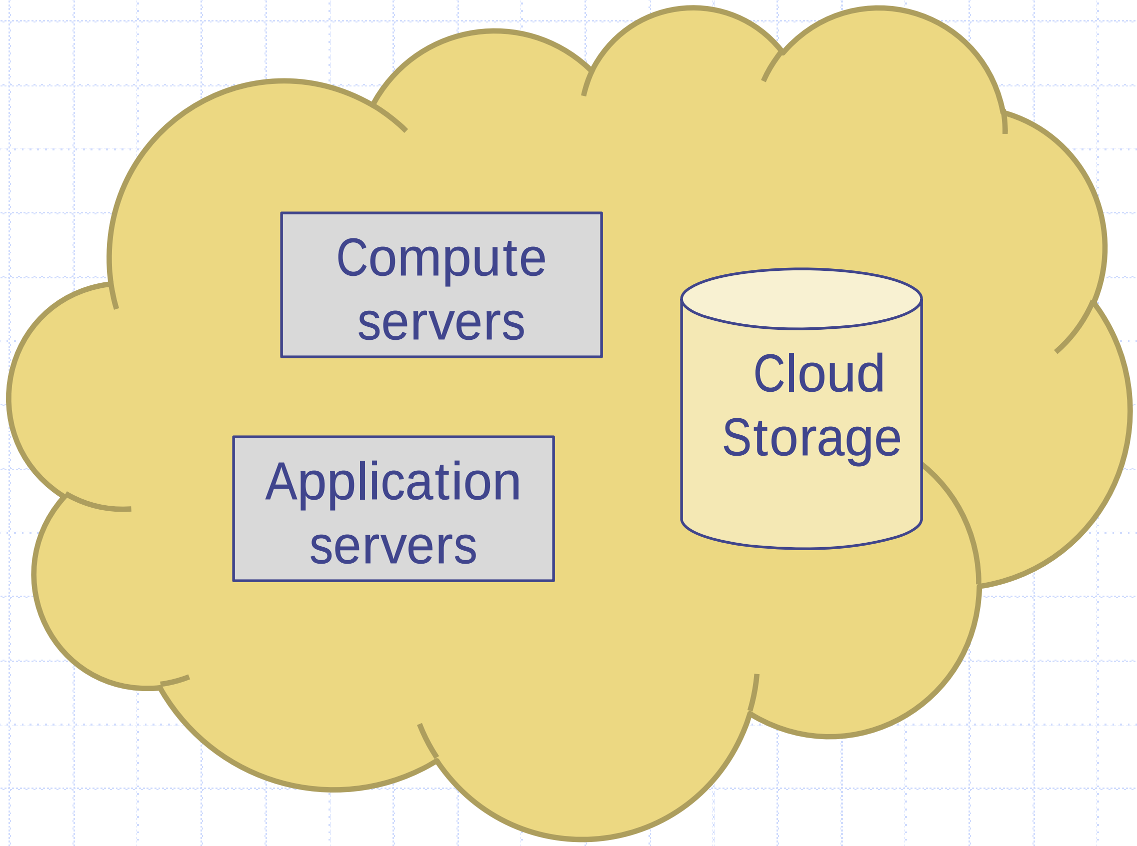
Dan Boneh, Kathleen Fisher,
John Mitchell

Challenge

- Cryptography gives us great tools
  - Secure function evaluation
  - Secret sharing
  - Homomorphic encryption
- How do we make these tools useful to a wider community of software developers?

Cloud computing

?



Specific challenge

- Can we send computation to the cloud, without revealing program or data?

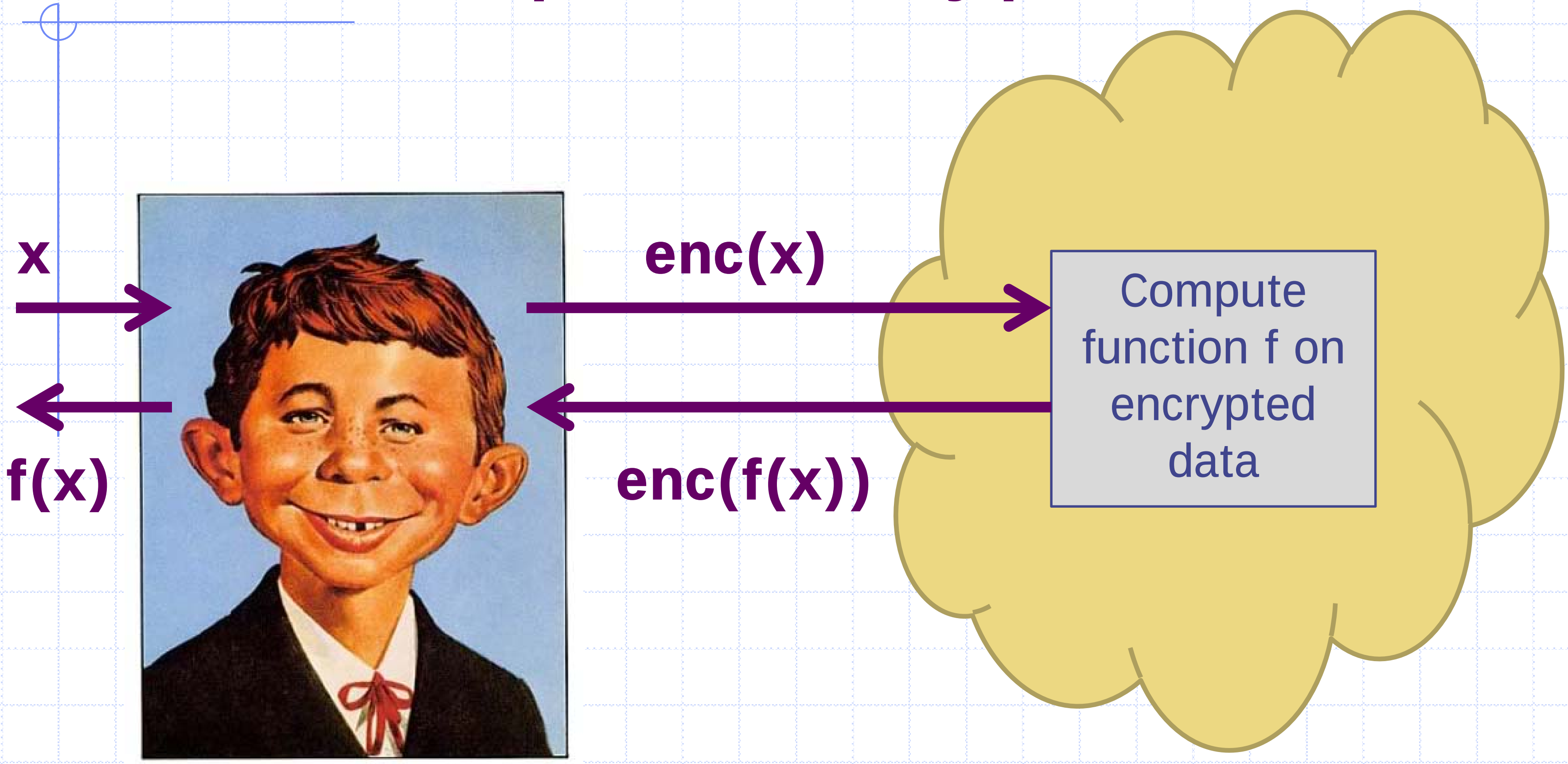
Specific challenge

- Can we send computation to the cloud, without revealing program or data?

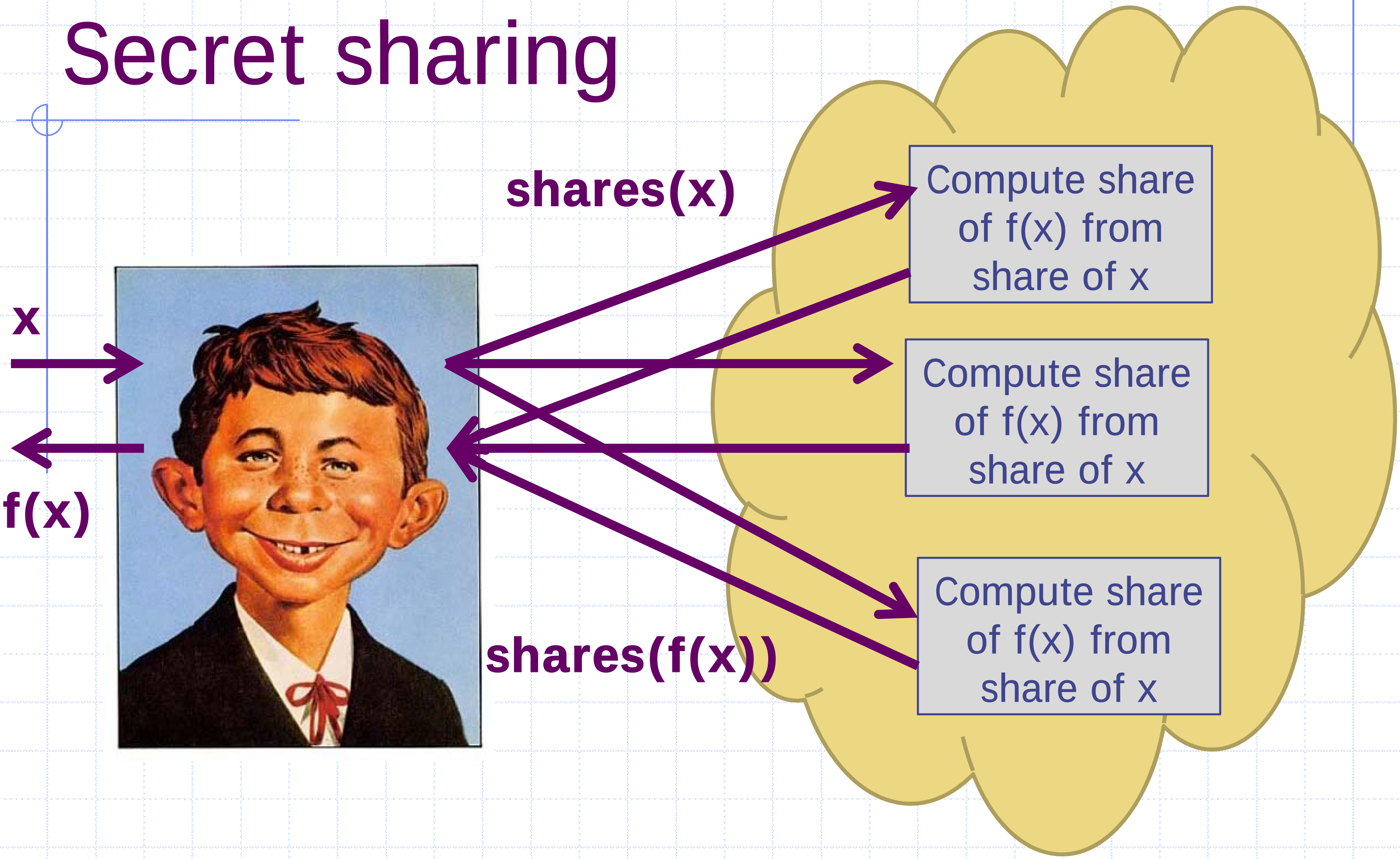
- Helpful ideas
  - Trusted local computation + untrusted cloud
    - Trusted preprocessing
    - Trusted post-processing
  - Suffices to solve the problem for *data*
    - Program can be “Universal Turing Machine”
      = Interpreter that receives ⟨program, input⟩

    (but consider other programs as examples too)

Homomorphic encryption



Secret sharing

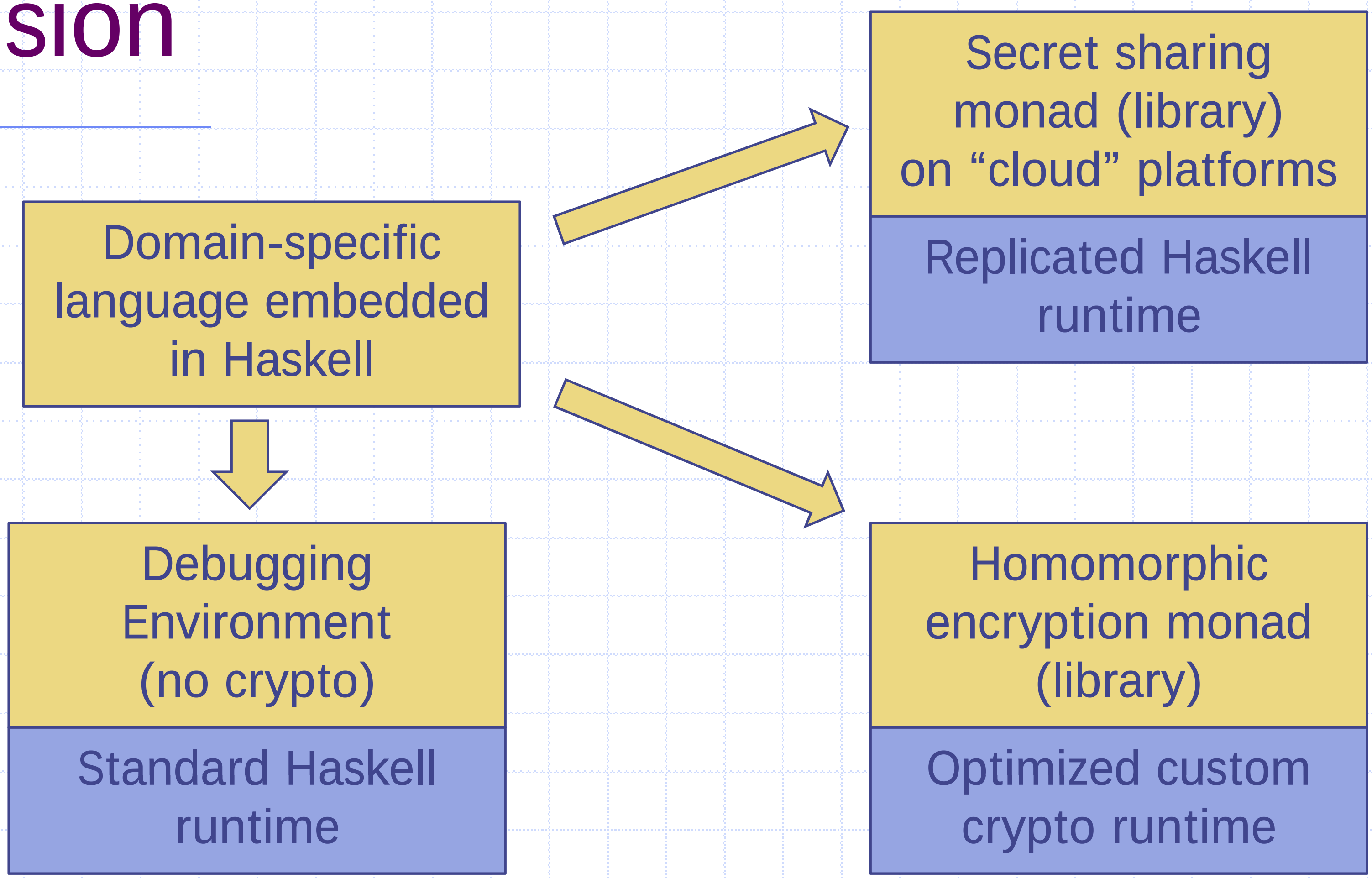


For n,k-secret sharing, secure if <k servers collude

Basic software approach

- Develop Domain Specific Language (DSL)
  - Based on Haskell, pure functional language
- Support code that allows *later* crypto choice
  - Homomorphic encryption
  - Secret sharing
- Key technical concept that gets us going
  - Haskell monads
  - Homomorphic encryption, secret sharing are both instances of monads, with similar operations

Vision



- Write Haskell code once using generic monad
- Execute code later on chosen platform

Haskell

- Haskell is a programming language that is
  - Functional: general-purpose, strongly typed, higher-order, type inference, interactive and compiled use
  - Lazy: purely functional core, extensible w/ effects
- Designed by committee in 80's and 90's to unify research efforts in lazy languages.
  - Haskell 1.0 in 1990, Haskell '98, Haskell ongoing

Higher-Order Functions

- Functions that take other functions as arguments or return a function as a result
- Common Examples:
  - Map: applies argument function to each element in a collection
  - Reduce: takes a collection, an initial value, and a function, and combines the elements in the collection according to function.

```
list = [1,2,3]
r = foldl (\accumulator i -> i + accumulator) 0 list
```

- Google uses Map/Reduce to parallelize and distribute massive data processing tasks [Dean, Ghemawat, OSDI 2004]
  (Haskell had these functional programming concepts long before Google)

Monads

- General concept from category theory
  - Adopted in Haskell for I/O, side effects, ...
- A monad consists of:
  - A type constructor M
  - A function **bind** :: M a -> (a -> M b) -> M b
  - A function **return** :: a -> M a
- Plus:
  - Laws about how these operations interact

Monad Examples

- Error handling $M(A) = A \cup \{error\}$
  - Add a special "error value" to a type
  - Define sequencing operator ";" to propagate error
- Information-flow tracking $M(A) = A \times Labels$
  - Add information flow label to each value
  - Define ";" to check and propagate labels
- Homomorphic encryption $M(A) = HomEnc(A)$
  - Represent values by encrypted values
  - Define ";" to homomorphically apply next function
- Secret sharing $M(A) = Shares(A)$
  - Represent value by list of shares
  - Define ";" to apply next function to shares

Can write code to compute on A, but run it using M(A)

Monad “do” and “bind” notation

- The special notation

```
do {v1 <- e1; e2}
```

is “syntactic” sugar for the ordinary expression

```
e1 >>= \v1 -> e2
```

where **>>=** (called bind) sequences actions

```
(>>=) :: M a -> (a -> M b) -> M b
```

- The value returned by the first action needs to be passed to the second; hence the 2nd arg to **>>=** is a function (often an explicit lambda).

Monad feature of Haskell

- Define monad for each type of encrypted data
  - Secret sharing, executed on independent platforms
  - Homomorphic encryption
  - “Identity” monad with no encryption (for testing)
- Conventional imperative notation
  - Haskell code for computing over monadic values looks like standard imperative code
- Interpreted using operations of monad
  - The sequence operator “;” of the monad composes functions on encrypted data

Homomorphic encryption monad

- Homomorphic encryption provides
  - For all encryptable types S and T, a map
    $(S \rightarrow T) \rightarrow \mathrm{Encrypt}(S,k) \rightarrow \mathrm{Encrypt}(T,k)$
    that allows a function on public data to be applied to encrypted data, producing encrypted results
- Haskell monadic programming requires
  - For all types S and T, a map
    $\mathrm{Encrypt}(S,k) \rightarrow (S \rightarrow \mathrm{Encrypt}(T,k)) \rightarrow \mathrm{Encrypt}(T,k)$
    that is used as the “;” for programming
- Mismatch
  - Resolved using circular-secure encryption (next slide)

Circular-secure encryption

- Proxy re-encryption

  $\text{Encrypt}(S,k_1) \to \text{Encrypt}(\text{Encrypt}(S,k_1),k_2)$
  $\to \text{Encrypt}(S,k_2)$

- Circular-secure proxy re-encryption

  $\text{Encrypt}(\text{Encrypt}(S,k),k) \to \text{Encrypt}(S,k)$

- Homomorphic encryption provides

  $\text{Encrypt}(S,k) \to (S \to \text{Encrypt}(T,k))$
  $\to \text{Encrypt}(\text{Encrypt}(T,k),k)$

- Monad condition satisfied
  - Compose result from homomorphism property with map associated with proxy re-encryption

Additional technical issue

- Homomorphic encryption provides
  - For all *encryptable* types S and T, ...
- Haskell monadic programming requires
  - For all types S and T, ...
- Problem
  - Haskell assumes every type can be monadic
  - Encryption applies only to numbers, pairs, ...
  - Homomorphism defined for functions representable by circuits
  - How do we resolve this without complicating the design and use of the programming language?

Some building blocks

- Information-flow analysis
  - Functions on encrypted data must not leak confidential values through control flow, i.e., language must prevent implicit information flow
  - There has been considerable research on static and dynamic information flow analysis
- Language-based computational complexity
  - Functions on encrypted data must terminate and in some cases must have poly-size circuits
  - S. Cook and students have produced language-based characterizations of complexity classes that yield type systems characterizing polynomial time

Current activities

- Define core expression language
  - Two types of integers: secret, public
  - Operations: add, multiply, if-then-else, ...
- Provide two semantics
  - Trusted sequential execution
  - Distributed execution on shares of secrets
- State and prove basic results
  - Compare sequential and distributed execution

Basic theorems (in progress)

- Expressiveness
  - For any computable function f: Int $\rightarrow$ Int, with computable time bound t, there is a program P : PInt $\times$ SInt $\rightarrow$ SInt with $P(t(|x|), x) = f(x)$
- Secrecy
  - At each step in any distributed computation on shares of a secret input, each node has learned only shares of the secret intermediate results computed by the corresponding centralized trusted computation

Summary

- Exciting crypto possibilities
  - Homomorphic encryption
  - Secret sharing
- Current work on languages, tools for programming on encrypted data
- Leverage
  - Functional programming, monad concept
  - Program semantics, equivalence proofs
  - Related work on secure multiparty computation, crypto programming languages, information flow, ...