Map-Based Exploration of Intrinsic Shape Differences and Variability

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Motivation

vs.

vs.
Motivation

\[ D = C + (B - A) \]
Goal

Descriptor of shape difference as seen by a given map:

- Where & how
- Global & local
- Manipulable
- Transferrable

\[ D = C + (B - A) \]
Maps can be used to transport tangent vectors
Classical Approach to Relating Shapes

To measure distortions induced by a map, track how inner products of vectors change after transporting.

Challenges:
- Point-wise information only
- Noisy
- Hard to aggregate
Challenge: new types of maps

Fuzzy maps
Kim et al., SIGGRAPH 12

Soft maps
Solomon et al., SGP’12

Functional maps
Ovsjanikov et al., SIGGRAPH’12

Meaningful p2p map?
Fuzzy map!
Maps (p2p, soft, fuzzy) can be used to transport functions.
Our approach

To measure distortions induced by a map, track how inner products of *vectors* change after transporting.
Outline

• Introduction
• Shape Differences
• Applications and Results:
  – Intrinsic shape space
  – Localized comparisons
  – Shape analogies
Input: Functional map $F$

$F$ is a linear operator (matrix)

$F : L^2(\text{cat}) \rightarrow L^2(\text{lion})$

Functions on cat are transferred to lion using $F$
Output: Shape differences

**V** – area-based shape difference

\[ V : L^2(cat) \rightarrow L^2(cat) \]

**R** – conformal shape difference

\[ R : L^2(cat) \rightarrow L^2(cat) \]
Measurement Discrepancies

\[ \int_{\text{lion}} F(f)F(g) \neq \int_{\text{cat}} fg \]

after \hspace{1cm} before
Observation

\[ \int_{\text{cat}} f g = \langle f, g \rangle_{\text{before}} \quad \int_{\text{lion}} F(f)F(g) = \langle f, g \rangle_{\text{after}} \]

are **both** inner products on \( L^2(\text{cat}) \).
The Universal Compensator

Riesz Representation Theorem

There exists a linear operator

\[ V : L^2(\text{cat}) \to L^2(\text{cat}) \]

such that

\[ \langle f, g \rangle_{\text{after}} = \langle f, V(g) \rangle_{\text{before}} \]
Area-based Shape Difference: $V$

\[
\int_{\text{lion}} F(f)F(g) \neq \int_{\text{cat}} fg \quad \Rightarrow \quad \int_{\text{lion}} F(f)F(g) = \int_{\text{cat}} fV(g)
\]
Area-based Shape Difference: $V$

\[
\int_N F(f)F(g) = \int_M fV(g)
\]
Consider a different inner-product of functions ... get information about conformal distortion

\[
\int_N \nabla F(f) \nabla F(g) = \int_M \nabla f \nabla R(g)
\]
APPLICATIONS AND RESULTS
Shape Differences in Collections

\[ D_1 \xrightarrow{F_1} N_1 \]

\[ D_2 \xrightarrow{F_2} N_2 \]

\[ M \]
Intrinsic shape space

Area

Conformal

2nd Principal Component (49.4%)

2nd Principal Component (49.8%)

1st Principal Component (50.6%)

D_2

M

F_1

N_1

D_1

F_2

N_2

...
Intrinsic shape space

13

Conformal

21

Area

16

24
Localized comparisons

\[ \rho : M \rightarrow \mathbb{R} \]

supported in ROI

\[ D_1 \rho \text{ to } D_2 \rho \]
Exaggeration of difference in ROI

Rest | ROI | Output: magnified distortion at ROI
Interpolation between poses along ROI

Initial | Output: interpolating poses on ROI | Final
Shape Differences in Collections

\[ D_{M,N} \sim C^{-1} D_{P,Q} C \]
Analogies: $D$ relates to $C$ as $B$ relates to $A$

$D = C + (B - A)$

hands raised up
Analogies: D relates to C as B relates to A
Shape Differences in Collections

\[ D_{M,N} \sim C^{-1} D_{P,Q} C \]

\[ \text{Spec}(D_{M,N}) \sim \text{Spec}(D_{P,Q}) \]
Aligning Collections

Complete graph

Complete graph
No cross map $F$ – used eigenvalues to compare shape differences
Shape Differences as Linear Operators
Thank you!

• Hao Li for the face models
• Keenan Crane for the bunny sequence
• Max Wardetzky and Henrik Schumacher for helping with Theorem 1
• Vladimir Kim for PPT images
• INRIA Associated team COMET
• CNRS chaire d'excellence
Thank you!

\[ C^{-1} \cdot C = \text{apple} \]