Achievable Rate Regions for the Broadcast Channel With Cognitive Relays

Jinhua Jiang, Ivana Marić, Andrea Goldsmith, and Shuguang Cui

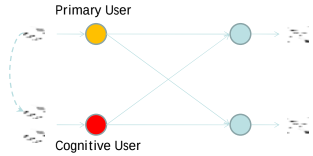
Summary

- We investigate the broadcast channel with two cognitive relays (BCCR) from an information theoretic perspective
  - new coding schemes based on rate splitting, Gel'fand-Pinsker coding, Marton's binning, and superposition coding are proposed
  - new achievable rate regions are derived, and are shown to include the existing ones
- Specialized one of achievable rate regions to the case with a single cognitive relay
  - a new achievable rate region is obtained for the well-studied cognitive radio channel (CRC)
- Developed a Gaussian example to demonstrate the dominance of the proposed coding schemes over other existing coding schemes

Motivations

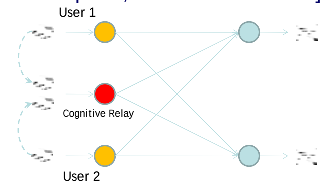
Two Simple network models involving CRs have been separately studied in the literature

- Cognitive radio channel (also called the interference channel with one cognitive transmitter) [Maric *et al.* 06', etc.]



- Most of the existing works on CRC view the channel as a variant of the interference channel
  - Most existing coding strategies are constrained by an encoding order: 1) primary user massage first, 2) then the cognitive user message with the Gel'fand-Pinsker coding scheme

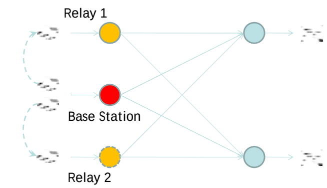
- Interference channel with a cognitive relay [sahin & Erkip 07'; Sridharan *et al.* 08']

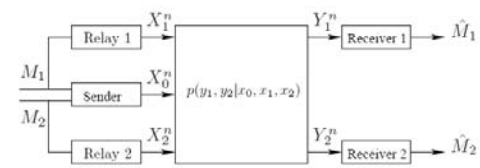


- Only the Gaussian case has been studied
- The generality of the channel model is largely ignored
  - The channel model in fact includes the CRC as a special case

Base on the observations, a unified approach is employed to study the two channels, which bridges the two channels with the same coding scheme, but breaks the constraints on the encoding sequence especially for the CRC!

Channel Model (BCCR)





Messages:

Encoding:

Decoding:

New Coding scheme

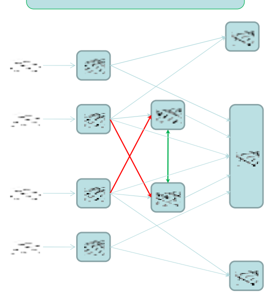
Rate splitting:

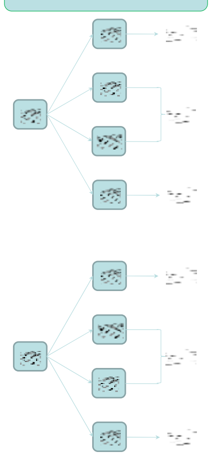
Joint PDF of the RVs for codebook generation:

(1)

Encoding

Decoding





Gel'fand-Pinsker Binning

Marton Binning

New Achievable Rate Regions

Thm. 1: For a fix joint PDF in (1), any non-negative rate pair $(R_{12}+R_{11}, R_{21}+R_{22})$ is achievable for the BCCR with R_{12}, R_{11}, R_{21}, and R_{22} satisfying

$$R_{11} - L_{11} + R_{22} - L_{22} \le -(I(W_1; V_2|V_1) + I(W_2; V_1|V_2) + I(W_2; W_1|V_1, V_2)),$$
$$R_{12} \le I(U_1; Y_1|V_1, W_1, U_2),$$
$$R_{12} + L_{11} - R_{11} \le I(U_1, W_1; Y_1|V_1, U_2),$$
$$L_{11} \le I(V_1, W_1; Y_1|U_1, U_2),$$
$$R_{12} + L_{11} \le I(U_1, V_1, W_1; Y_1|U_2),$$
$$L_{11} + R_{21} \le I(V_1, W_1, U_2; Y_1|U_1),$$
$$R_{12} + R_{21} \le I(U_1, U_2; Y_1|V_1, W_1),$$
$$R_{12} + R_{21} + L_{11} - R_{11} \le I(U_1, U_2, W_1; Y_1|V_1),$$
$$R_{12} + L_{11} + R_{21} \le I(U_1, V_1, W_1, U_2; Y_1),$$
$$R_{21} \le I(U_2; Y_2|V_2, W_2, U_1),$$
$$R_{21} + L_{22} - R_{22} \le I(U_2, W_2; Y_2|V_2, U_1),$$
$$L_{22} \le I(V_2, W_2; Y_2|U_2, U_1),$$
$$R_{21} + L_{22} \le I(U_2, V_2, W_2; Y_2|U_1),$$
$$L_{22} + R_{12} \le I(V_2, W_2, U_1; Y_2|U_2),$$
$$R_{21} + R_{12} \le I(U_2, U_1; Y_2|V_2, W_2),$$
$$R_{21} + R_{12} + L_{22} - R_{22} \le I(U_2, U_1, W_2; Y_2|V_2),$$
$$R_{21} + L_{22} + R_{12} \le I(U_2, V_2, W_2, U_1; Y_2),$$

Where $L_{11} >= R_{11}$, and $L_{22} >= R_{22}$.

- This new rate region is shown to include the best existing rate region for the BCCR in [Sridharan *et al.* 08']
- A further improved rate region with simplified description is obtained using successive superposition encoding with the following joint PDF:

(2)

Thm. 2: For a fix joint PDF in (2), any non-negative rate pair $(R_{12}+R_{11}, R_{21}+R_{22})$ is achievable for the BCCR with R_{12}, R_{11}, R_{21}, and R_{22} satisfying

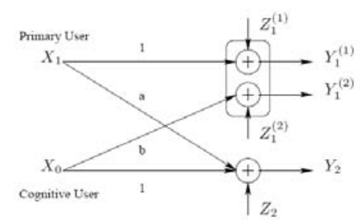
$$R_{11} - L_{11} + R_{22} - L_{22} \le -(I(W_1; V_2|V_1, U_1, U_2) + I(W_2; V_1|V_2, U_1, U_2) + I(W_2; W_1|V_1, V_2, U_1, U_2)),$$
$$L_{11} \le I(V_1, W_1; Y_1|U_1, U_2),$$
$$R_{12} + L_{11} \le I(U_1, V_1, W_1; Y_1|U_2),$$
$$L_{11} + R_{21} \le I(V_1, W_1, U_2; Y_1|U_1),$$
$$R_{12} + L_{11} + R_{21} \le I(U_1, V_1, W_1, U_2; Y_1),$$
$$L_{22} \le I(V_2, W_2; Y_2|U_2, U_1),$$
$$R_{21} + L_{22} \le I(U_2, V_2, W_2; Y_2|U_1),$$
$$L_{22} + R_{12} \le I(V_2, W_2, U_1; Y_2|U_2),$$
$$R_{21} + L_{22} + R_{12} \le I(U_2, V_2, W_2, U_1; Y_2).$$

Set V_2 as a constant in (2):

Thm. 3: Any non-negative rate pair $(R_{12}+R_{11}, R_{21}+R_{22})$ is achievable for the CRC with R_{12}, R_{11}, R_{21}, and R_{22} satisfying

$$R_{11} - L_{11} + R_{22} - L_{22} \le -I(W_2; W_1, V_1|U_1, U_2),$$
$$L_{11} \le I(V_1, W_1; Y_1|U_1, U_2),$$
$$R_{12} + L_{11} \le I(U_1, V_1, W_1; Y_1|U_2),$$
$$L_{11} + R_{21} \le I(V_1, W_1, U_2; Y_1|U_1),$$
$$R_{12} + L_{11} + R_{21} \le I(U_1, V_1, W_1, U_2; Y_1),$$
$$L_{22} \le I(W_2; Y_2|U_2, U_1),$$
$$R_{21} + L_{22} \le I(U_2, W_2; Y_2|U_1),$$
$$L_{22} + R_{12} \le I(W_2, U_1; Y_2|U_2),$$
$$R_{21} + L_{22} + R_{12} \le I(U_2, W_2, U_1; Y_2).$$

A Gaussian Example



$Y_1^{(1)} = X_1 + Z_1^{(1)}$, $Y_1^{(2)} = bX_0 + Z_1^{(2)}$,
$Y_2 = X_0 + aX_1 + Z_2$.

$b > 1$

Set V_2, U_1 and U_2 as constants in (2):

$$R_1 \le I(V_1, W_1; Y_1),$$
$$R_2 \le I(W_2; Y_2),$$
$$R_1 + R_2 \le I(V_1, W_1; Y_1) + I(W_2; Y_2) - I(W_2; W_1, V_1).$$

Evaluate

$$R_1 \le \frac{1}{2}\log_2(1+P_1) + \frac{1}{2}\log_2(1+(1-\alpha)b^2 P_0),$$
$$R_2 \le \frac{1}{2}\log_2\left(1+\frac{\alpha P_0}{(1-\alpha)P_0+1}\right).$$

- Dirty paper coding is applied twice in a unique order: encode W_2 against V_1, and then encode W_1 against W_2, which is impossible with any other existing coding schemes for the CRC or the BCCR
- The obtained rate region is the capacity region for the channel when the interference link from X_1 to Y_2 is removed

Future work: 1) to determine whether the above region is the capacity region for the example setting; 2) also for the general Gaussian setting.