Program Trading: Agency vs. Principal

Program trading, the buying/selling of large portfolios, operates in two formats:

- **Agency**:
  - Broker executes trades on behalf of the client (VWAP is a common target).
  - All risk is held by the *client*.
  - Client pays a fixed commission.

- **Principal**:
  - Client sells its portfolio to the broker at a mutually agreed upon spot price plus commission.
  - All risk is transferred to the *broker*.
  - Result of a preliminary **blind basket auction**. Accounts for roughly 12% of daily shares traded on NYSE (18 billion dollars) daily.
Blind Basket Auction

- Seller contacts a small number of brokers and provides each with a description of the portfolio’s characteristics.
- Exact identities of names in portfolio are not revealed.
- Seller decides whether to sell entire portfolio to broker with best bid.
Inefficiencies

There are several areas of inefficiency...

1. **The uncertainty about the portfolio’s contents reduces the valuations of risk-averse buyers.** Due to the blind nature of the bidding process, brokers are concerned about adverse selection effects.

2. **It is desirable to solicit bids from a larger number of potential buyers.** Due to the information leakage that occurs in the current blind auction mechanism, sellers want to limit their bid solicitation to a very limited number of brokers.

3. **The value of the portfolio may be increased through division into parts sold to multiple bidders.** It is likely that different pieces of the portfolio are valued more by different brokers.

We focus here on items 1 and 2.
Objective

Address points 1 and 2 by introducing an intermediary between the seller and brokers.

Goal: To show via a computational study based on a game-theoretic model the significant benefit to seller transaction costs possible if the auction is intermediated.
Model

A single risk-neutral seller and N risk-averse bidders.

Assumption 1 - Brokers’ Utilities

Brokers exhibit constant absolute risk aversion. In other words, there exists a scalar \( r > 0 \) such that \( u(v) = -\exp(-rv) \) for all \( v \).

The role of private information is central to our problem and arises through the dollar value \( v_n \) of the portfolio to each broker.

Assumption 2 - Brokers’ Valuations

For each \( n \), the value of the portfolio to the \( n \)th broker is given by \( v_n = v^* - q\theta_n \), where \( \theta_1, \ldots, \theta_N \) are iid random variables independent from \( q \). Further, the seller only observes \( q \) and each \( n \)th broker only observes \( \theta_n \).

\( v^* \): nominal portfolio value (common knowledge), \( q \): portfolio characteristic (seller knowledge), \( \theta_n \): broker type (\( \sim F \)) (broker knowledge)
Standard First-Price Auction I

Given $N$, $r$, and $v^*$, we assume brokers employ a symmetric strategy $\beta_s$ mapping $\theta_n$ to a bid $\beta_s(\theta_n)$. As a model of broker behavior, we consider a bidding strategy that forms part of a Bayesian-Nash equilibrium.

Lemma 1 - Unique Symmetric Bayesian-Nash Bidding Strategy

Let Assumptions 1 and 2 hold. There exists a unique symmetric Bayesian-Nash equilibrium bidding function for the standard first-price auction. This bidding function is strictly decreasing in $\theta$ and is differentiable.

As such, this bidding function $\beta_S$ uniquely satisfies the following expression

$$\beta_S(\theta_n) \in \arg\max_{b \in \mathbb{R}} E \left[ u \left( (v_n - b) 1(b \geq \max_{m \neq n} \beta_S(\theta_m)) \right) \mid \theta_n \right].$$
Standard First-Price Auction II

Solving for this gives us the unique symmetric bidding function.

**Theorem 1 - B-N Equilibrium Bids in a Standard First-Price Auction**

*Let Assumptions 1 and 2 hold. In a standard first-price auction, the unique symmetric Bayesian-Nash equilibrium bidding function is given by*

\[
\beta_S(\theta_n) = v_{ce}(\theta_n) + \frac{1}{r} \ln \left( 1 - \frac{re^{-rv_{ce}(\theta_n)}}{F_{ce}(v_{ce}(\theta_n))^{N-1}} \int_0^{v_{ce}(\theta_n)} F_{ce}(\rho)^{N-1} e^{\rho} d\rho \right)
\]

*In fact, this is the only Bayesian-Nash equilibrium...*

**Theorem 2 - Unique Bayesian-Nash Bidding Strategy**

*Let Assumptions 1 and 2 hold. In a standard first-price auction, there exists a unique Bayesian-Nash equilibrium bidding function.*
Intermediated First-Price Auction

Each broker supplies to the intermediary a function $\beta(\theta_n, \cdot, \cdot)$ of the portfolio parameter $q$ and $N$. As before, we model broker behavior by a Bayesian-Nash equilibrium bidding function (again unique), i.e.

$$\beta_I(\theta_n, q, N) \in \arg\max_{b \in \mathbb{R}} E \left[ u \left( (v_n - b) \mathbf{1}(b \geq \max_{m \neq n} \beta_I(\theta_m, q, N)) \right) \mid \theta_n, q, N \right].$$

This leads us to the associated bidding strategy...

Theorem 3 - B-N Equilibrium Bids in an Intermediated First-Price Auction

Let Assumptions 1 and 2 hold. In an intermediated first-price auction, the unique Bayesian-Nash equilibrium bidding function is given by

$$\beta_I(\theta_n, q, N) = v(\theta_n) + \frac{1}{r} \ln \left[ 1 - \frac{re^{-rv(\theta_n)}}{F_v(v(\theta_n))^{N-1}} \int_0^{v(\theta_n)} F_v(\rho)^{N-1} e^{r\rho} d\rho \right].$$
Computational Study - Metric & Representative Values

- We compared via Monte-Carlo the relative performance of standard and intermediated mechanisms regarding seller transaction cost, i.e.

\[ T = v^* - b_w, \]

where \( b_w \) is the winning bid. This is the difference between the portfolio’s “book value” and what they actually receive for it.

- Values:
  - \( v^* = 5 \times 10^8 \), representing a 500 million dollar portfolio.
  - \( q \sim \text{unif}[0, 10^7] \), and \( \theta_n \sim \text{unif}[0, 1] \), representing broker transaction costs of 0 to 10 million dollars.
  - \( N \in \{2, \ldots, 8\} \).
  - \( r \in [0, 4 \times 10^{-7}] \), capturing a realistic range of risk-aversion parameters (see paper).
Results - Seller Transaction Costs vs. Risk

- Num Brokers = 2
  - Intermediated
  - Standard

- Num Brokers = 4
  - Intermediated
  - Standard

- Num Brokers = 6
  - Intermediated
  - Standard

- Num Brokers = 8
  - Intermediated
  - Standard
Results - Summary

As $r$ increases, brokers face two opposing pressures:
- The optimal point where the marginal decrease in expected utility balances the marginal gain in winning probability will occur for a larger optimal bid, i.e. brokers tend to bid more.
- $v_{ce}$ for each broker decreases and brokers tend to bid less.

⇒ Degree of risk in portfolio will determine which factor dominates.

Gives us the following results:
- For all $r > 0$, $\bar{T}(I) < \bar{T}(S)$ and the gap between the two increases with $r$ (e.g. over 10% savings for $N = 2$).
- In the standard case, $\bar{T}(S)$ increases as $r$ increases.
- In the intermediated case, $\bar{T}(I)$ decreases as $r$ increases.

**Theorem 4 - Transaction Costs Decrease with $r$ with Intermediation**

(Sketch) Let Assumption 2 hold. If a utility $u_2$ is strictly more “risk averse” than another utility $u_1$, then $\bar{T}(I)$ is strictly less if brokers realize utility $u_2$ than if brokers realize utility $u_1$. 

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Summary

Summary...

- We have demonstrated via both computational and theoretical results the potential benefit to sellers of intermediation.
- Under reasonable assumptions of brokers’ utility functions and investment preferences (see paper), transaction cost improvements of over 10% are realized.
- The first investigation of how the notion of uncertainty affects blind portfolio auction efficiency.
Continuing Work

Current extensions of this work...

- Enhancing broker valuation model to account for the allocative inefficiency seen in practice.

- Empirical study of information leakage effects, and market efficiency (with C. Giannikos, CUNY).

- A repeated game-theoretic model to study reputation effects in bidding behavior (with C. Giannikos, CUNY).

- Consider a model with correlation amongst brokers’ types.

- Model/methods to award a portfolio across different brokers.
Appendix

These next slides illuminate in more detail targeted material from the previous discussion.
Theorem 4 : Costs Decrease with $r$ with Intermediation

Figure: Two utility functions, the more risk-averse one normalized in the second figure such that $u_2(0) = 0$ and $u_1(v - \beta(v)) = u_2(v - \beta(v))$. Here $v - \beta(v) = 1$ corresponds to the optimal bid $\beta(v)$ for the less risk-averse utility function.
Values for $r$

- Consider a broker who is uncertain about what a portfolio is worth to him and assumes a normal distribution with a standard deviation of one million dollars around his expectation.

- What premium would have to be subtracted from his expectation to arrive at a price where he is indifferent about acquiring the portfolio?

- A representative figure might be one hundred thousand dollars, which is ten percent of the standard deviation → a risk aversion of $r = 2 \times 10^{-7}$.

- E.g., letting the broker’s profit, which is the difference between the amount he pays for the portfolio and the amount he later discovers it is worth to him, be denoted by $x \sim N(10^5, (10^6)^2)$, this value of $r$ solves

$$v_{CE}(\theta_n) = -\frac{1}{r} \ln \left( -E \left[ -e^{-rx} \right] \right) = 10^5 - \frac{r}{2}(10^6)^2 = 0.$$
Model to Account for Allocative Inefficiency

- Here we use
  \[ v_n = v^* + f(\theta_n q - 1), \]
  where \( \theta_n \sim \text{unif}[-1, 1] \), \( q \sim \text{unif}[-1, 1] \), and \( f \in \mathbb{R}^+ \). Results in \( v_n \in [v^* - 2f, v^*] \).

- Other distributions also lead to closed-form solutions.