Blind Portfolio Auctions via Intermediaries

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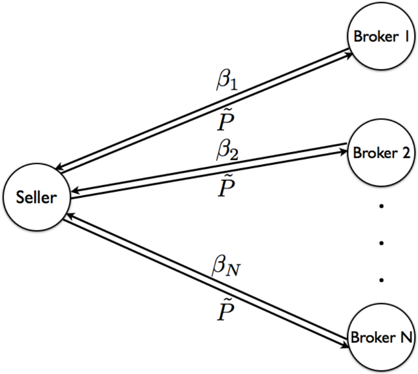
Program Trading: Agency vs. Principal

Program trading, the buying/selling of large portfolios, operates in two formats:

- **Agency**:
  - Broker executes trades on behalf of the client (VWAP is a common target).
  - All risk is held by the *client*.
  - Client pays a fixed commission.
- **Principal**:
  - Client sells its portfolio to the broker at a mutually agreed upon spot price plus commission.
  - All risk is transferred to the *broker*.
  - Result of a preliminary **blind basket auction**. Accounts for roughly 12% of daily shares traded on NYSE (18 billion dollars) daily.

Blind Basket Auction

- Seller contacts a small number of brokers and provides each with a description of the portfolio's characteristics.
- Exact identities of names in portfolio are not revealed.
- Seller decides whether to sell entire portfolio to broker with best bid.



Inefficiencies

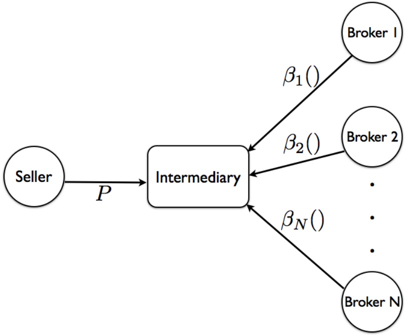
There are several areas of inefficiency...

1. **The uncertainty about the portfolio's contents reduces the valuations of risk-averse buyers**. Due to the blind nature of the bidding process, brokers are concerned about adverse selection effects.
2. **It is desirable to solicit bids from a larger number of potential buyers.** Due to the information leakage that occurs in the current blind auction mechanism, sellers want to limit their bid solicitation to a very limited number of brokers.
3. **The value of the portfolio may be increased through division into parts sold to multiple bidders.** It is likely that different pieces of the portfolio are valued more by different brokers.

We focus here on items 1 and 2.

Objective

Address points 1 and 2 by introducing an intermediary between the seller and brokers.



Goal: To show via a computational study based on a game-theoretic model the significant benefit to seller transaction costs possible if the auction is intermediated.

Model

A single risk-neutral seller and N risk-averse bidders.

Assumption 1 - Brokers' Utilities

Brokers exhibit constant absolute risk aversion. In other words, there exists a scalar $r > 0$ *such that* $u(v) = -\exp(-rv)$ *for all* v.

The role of private information is central to our problem and arises through the dollar value v_n of the portfolio to each broker.

Assumption 2 - Brokers' Valuations

For each n*, the value of the portfolio to the nth broker is given by* $v_n = v^* - q\theta_n$*, where* $\theta_1, \ldots, \theta_N$ *are iid random variables independent from* q*. Further, the seller only observes* q *and each nth broker only observes* θ_n*.*

v^* : nominal portfolio value (common knowledge), q : portfolio characteristic (seller knowledge), θ_n : broker type ($\sim F$) (broker knowledge)

Standard First-Price Auction I

Given N, r, and v^*, we assume brokers employ a symmetric strategy β_s mapping θ_n to a bid $\beta_s(\theta_n)$. As a model of broker behavior, we consider a bidding strategy that forms part of a Bayesian-Nash equilibrium.

Lemma 1 - Unique Symmetric Bayesian-Nash Bidding Strategy

Let Assumptions 1 and 2 hold. There exists a unique symmetric Bayesian-Nash equilibrium bidding function for the standard first-price auction. This bidding function is strictly decreasing in θ and is differentiable.

As such, this bidding function β_{S} uniquely satisfies the following expression

$$\beta_{\mathrm{S}}(\theta_n) \in \underset{b \in \Re}{\operatorname{argmax}}\, E\left[u\left((v_n - b)\mathbf{1}(b \geq \max_{m \neq n} \beta_{\mathrm{S}}(\theta_m))\right) \,\middle|\, \theta_n\right].$$

Standard First-Price Auction II

Solving for this gives us the unique symmetric bidding function.

Theorem 1 - B-N Equilibrium Bids in a Standard First-Price Auction

Let Assumptions 1 and 2 hold. In a standard first-price auction, the unique symmetric Bayesian-Nash equilibrium bidding function is given by

$$\beta_{\mathrm{S}}(\theta_n) = v_{ce}(\theta_n) + \frac{1}{r}\ln\left[1 - \frac{re^{-rv_{ce}(\theta_n)}}{F_{ce}(v_{ce}(\theta_n))^{N-1}}\int_0^{v_{ce}(\theta_n)} F_{ce}(\rho)^{N-1}e^{r\rho}d\rho\right]$$

In fact, this is the only Bayesian-Nash equilibrium...

Theorem 2 - Unique Bayesian-Nash Bidding Strategy

Let Assumptions 1 and 2 hold. In a standard first-price auction, there exists a **unique** *Bayesian-Nash equilibrium bidding function.*

Intermediated First-Price Auction

Each broker supplies to the intermediary a function $\beta(\theta_n, \cdot, \cdot)$ of the portfolio parameter q and N. As before, we model broker behavior by a Bayesian-Nash equilibrium bidding function (again unique), i.e.

$$\beta_I(\theta_n, q, N) \in \underset{b \in \Re}{\operatorname{argmax}} \, E\left[u\left((v_n - b)\mathbf{1}(b \geq \max_{m \neq n} \beta_I(\theta_m, q, N)) \right) \;\middle|\; \theta_n, q, N \right].$$

This leads us to the associated bidding strategy...

Theorem 3 - B-N Equilibrium Bids in an Intermediated First-Price Auction

Let Assumptions 1 and 2 hold. In an intermediated first-price auction, the unique Bayesian-Nash equilibrium bidding function is given by

$$\beta_I(\theta_n, q, N) = v(\theta_n) + \frac{1}{r} \ln \left[1 - \frac{r e^{-r v(\theta_n)}}{F_v(v(\theta_n))^{N-1}} \int_0^{v(\theta_n)} F_v(\rho)^{N-1} e^{r\rho} d\rho \right]$$

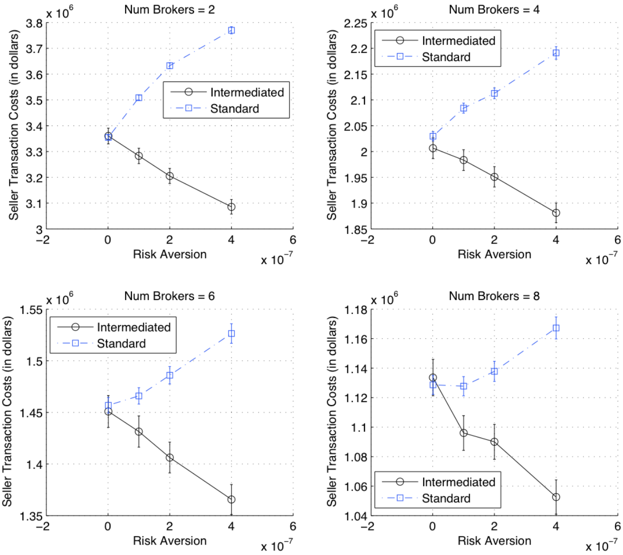
Computational Study - Metric & Representative Values

- We compared via Monte-Carlo the relative performance of standard and intermediated mechanisms regarding seller transaction cost, i.e.

$$T = v^* - b_w,$$

  where b_w is the winning bid. This is the difference between the portfolio's "book value" and what they actually receive for it.
- Values:
  - $v^* = 5 \times 10^8$, representing a 500 million dollar portfolio.
  - $q \sim \mathrm{unif}[0, 10^7]$, and $\theta_n \sim \mathrm{unif}[0, 1]$, representing broker transaction costs of 0 to 10 million dollars.
  - $N \in \{2, \ldots, 8\}$.
  - $r \in [0, 4 \times 10^{-7}]$, capturing a realistic range of risk-aversion parameters (see paper).

Results - Seller Transaction Costs vs. Risk



Results - Summary

As r increases, brokers face two opposing pressures:

- The optimal point where the marginal decrease in expected utility balances the marginal gain in winning probability will occur for a larger optimal bid, i.e. brokers tend to bid more.
- v_{ce} for each broker decreases and brokers tend to bid less.

$\Rightarrow$ Degree of risk in portfolio will determine which factor dominates.

Gives us the following results:

- For all $r > 0$, $\bar{T}(I) < \bar{T}(S)$ and the gap between the two increases with r (e.g. over 10% savings for $N = 2$).
- In the standard case, $\bar{T}(S)$ increases as r increases.
- In the intermediated case, $\bar{T}(I)$ decreases as r increases.

Theorem 4 - Transaction Costs Decrease with r with Intermediation

(Sketch) Let Assumption 2 hold. If a utility u_2 is strictly more "risk averse" than another utility u_1, then $\bar{T}(I)$ is strictly less if brokers realize utility u_2 than if brokers realize utility u_1.

Summary

Summary...

- We have demonstrated via both computational and theoretical results the potential benefit to sellers of intermediation.
- Under reasonable assumptions of brokers' utility functions and investment preferences (see paper), transaction cost improvements of over 10% are realized.
- The first investigation of how the notion of uncertainty affects blind portfolio auction efficiency.

Continuing Work

Current extensions of this work...

- Enhancing broker valuation model to account for the allocative inefficiency seen in practice.
- Empirical study of information leakage effects, and market efficiency (with C. Giannikos, CUNY).
- A repeated game-theoretic model to study reputation effects in bidding behavior (with C. Giannikos, CUNY).
- Consider a model with correlation amongst brokers' types.
- Model/methods to award a portfolio across different brokers.

Appendix

These next slides illuminate in more detail targeted material from the previous discussion.

Theorem 4 : Costs Decrease with r with Intermediation

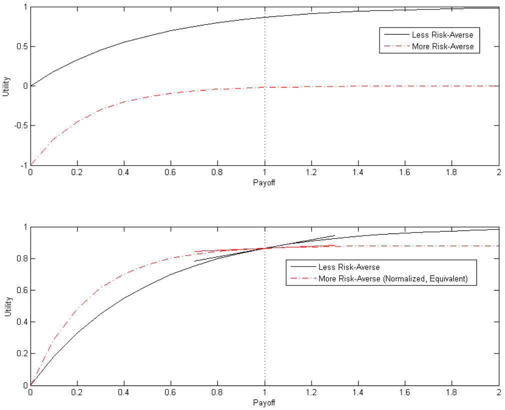


Figure: *Two utility functions, the more risk-averse one normalized in the second figure such that* $u_2(0) = 0$ *and* $u_1(v - \beta(v)) = u_2(v - \beta(v))$. *Here* $v - \beta(v) = 1$ *corresponds to the optimal bid* $\beta(v)$ *for the less risk-averse utility function.*

Values for r

- Consider a broker who is uncertain about what a portfolio is worth to him and assumes a normal distribution with a standard deviation of one million dollars around his expectation.
- What premium would have to be subtracted from his expectation to arrive at a price where he is indifferent about acquiring the portfolio?
- A representative figure might be one hundred thousand dollars, which is ten percent of the standard deviation $\rightarrow$ a risk aversion of $r = 2 \times 10^{-7}$.
- E.g., letting the broker's profit, which is the difference between the amount he pays for the portfolio and the amount he later discovers it is worth to him, be denoted by $x \sim \mathrm{N}(10^5, (10^6)^2)$, this value of r solves

$$v_{\mathrm{CE}}(\theta_n) = -\frac{1}{r} \ln \left(-E \left[-e^{-rx} \right] \right) = 10^5 - \frac{r}{2}(10^6)^2 = 0.$$

Model to Account for Allocative Inefficiency

- Here we use
$$v_n = v^* + f(\theta_n q - 1),$$
where $\theta_n \sim \text{unif}[-1, 1]$, $q \sim \text{unif}[-1, 1]$, and $f \in \Re^+$. Results in $v_n \in [v^* - 2f, v^*]$.

- Other distributions also lead to closed-form solutions.