

# Manipulation Robustness of Collaborative Filtering Systems

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# Abstract

- CF heuristics deployed in industry highly manipulable.
- *Linear* CF algorithms much more robust.
- We provide theoretical guarantee and empirical support.

# Preliminaries

- Introduction of CF systems.



## Customers Who Bought This Item Also Bought



[A Beautiful Mind](#)  
 (Widescreen Awards Edition)  
 DVD ~ Paul Bettany  
 ★★★★★ (167) \$9.99



[Finding Forrester](#) DVD ~  
 Sean Connery  
 ★★★★★ (217) \$6.99



[Bounders \(Collector's Edition\)](#) DVD ~ Matt Damon  
 ★★★★★ (165) \$11.49



[Rain Man \(Special Edition\)](#)  
 DVD ~ Tom Cruise  
 ★★★★★ (128) \$9.99

- Much online consumption driven by CF. [Anderson 06]



- Nearest Neighbor (NN) methods common.

# Manipulation

Manipulation likely frequent.

- Incentivized by revenue impact.
- Rampant manipulation in other recommendation systems.
- Example: Christianity and homosexuality on Amazon. [Olsen 2002]
- Method: inject ratings “typical” except on targets.

[Burke 2005]

# Model

- $N$  products.
- Types  $\in \bar{S}$ , ratings  $\in S = \bar{S} \cup \{?\}$ .
- $M$  ratings vectors, each  $\in S^N$ .
- Fraction of manipulated data  $r$ .
- Honest ratings vectors  $Y \in S^{N \times (1-r)M}$ .
- Manipulated ratings vectors  $Z \in S^{N \times rM}$ .
- Training set  $(Y, Z)$ .

# CF Algorithms

- Active user inspects and rates products  $\nu_1, \dots, \nu_n$ .
- CF algorithm maps  $\{1, \dots, N\} \times S^N \times S^{N \times M}$  to PMF over  $\bar{S}$ :

$$p_{\nu_k, x^{k-1}, W}.$$

- Scalar prediction  $\tilde{x}_{\nu_k}^W = E[x_{\nu_k}]$  w.r.t.  $p_{\nu_k, x^{k-1}, W}$ .
- RMS distortion:

$$d_n(p, \nu, Y, Z) = \sqrt{\frac{1}{n} \sum_{k=1}^n E \left[ \left( \tilde{x}_{\nu_k}^Y - \tilde{x}_{\nu_k}^{(Y, Z)} \right)^2 \right]}.$$

# Linear CF Algorithms

- Probabilistic CF algorithm  $p$ :

for each  $W$ , exists PMF  $\hat{\psi}^{p,W}$  over  $\bar{S}^N \times S^N$  for each  $n, x$ , s.t.

$$p_{n,x,W} = \hat{\psi}^{p,W}(\bar{x}_n|x).$$

- Linear CF algorithm  $p$ :

for any  $W_1 \in S^{N \times M_1}$  and  $W_2 \in S^{N \times M_2}$ ,

$$\hat{\psi}_{\bar{S}}^{p,(W_1,W_2)} = \frac{M_1}{M_1 + M_2} \hat{\psi}_{\bar{S}}^{p,W_1} + \frac{M_2}{M_1 + M_2} \hat{\psi}_{\bar{S}}^{p,W_2}.$$

# Results

## Theorem

$p$  is a linear CF algorithm. For all  $M, r, Y, Z$ , and  $\nu$ ,

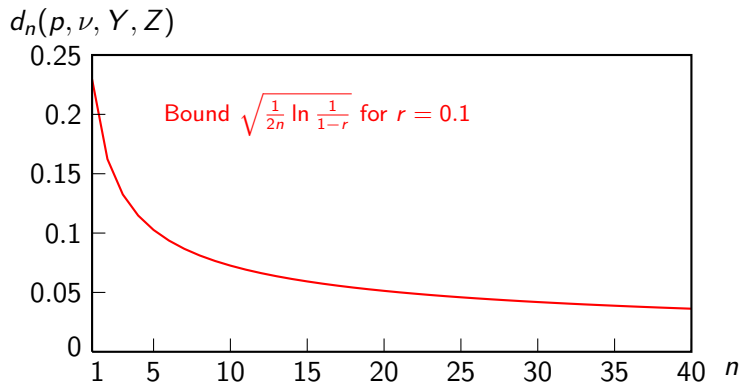
$$d_n(p, \nu, Y, Z) \leq \sqrt{\frac{1}{2n} \ln \frac{1}{1-r}}.$$

Intuition:

- $\hat{\psi}^{p,W}$  is convex combination of  $\hat{\psi}^{p,Y}$  and  $\hat{\psi}^{p,Z}$ .
- Weights on the latter decays as  $n$  increases.



# Practical Implications

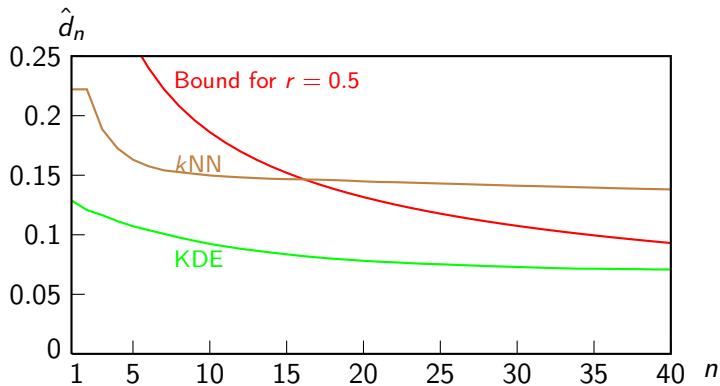


Binary setting:  $r \leq 0.1, n \geq 22 \Rightarrow$  % correct predictions  $\searrow \leq 0.05$ .

# Empirical Evaluation

- Netflix data: (user, movie, rating) tuples.
- 5K users, 500 movies, 200K ratings.
- 20% test data.
- Injected 50% manipulated ratings vectors. In each,
  - Half sampled from empirical distribution.
  - Half are 1's.
- Applied
  - Linear CF algorithm: kernel density estimation.
  - NN algorithm:  $k$ NN.

# Results



# Conclusion

- Theoretical and empirical analyses demonstrate that
  - NN algorithms highly manipulable.
  - Linear CF algorithms counter manipulation.
  - Users should rate a minimum number of products.
- Our framework and insights can
  - Help make existing CF systems more robust.
  - Help assess additional countermeasures.