Message Passing for Matrix Factorization

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Model

- Let \( N \in \mathbb{R}^{m \times n} \) be ‘approximately’ low rank
  \( N = UV^T + W \)
- A small subset \( E \) of entries revealed
- \( U \) and \( V \) are typically low rank
- Compute \( U^TV \) from the subset of entries revealed

Motivation

Recommendation Systems

- \( N \) is the matrix of user ratings
- \( N_{ij} \) is rating given by user \( i \) to product \( j \)
- We want to learn unrevealed \( N_{ij} \)
- Recommend product \( j \) to customer \( i \) if \( \hat{N}_{ij} \) is large

Sensor Localization

- \( N_{ij} = D_{ij}^2 : D_{ij} \) distance between sensors \( i \) and \( j \)
- Sensors can only detect nearby sensors
  - Only small \( N_{ij} \) are revealed
  - Matrix \( \{D_{ij}^2\} \) has rank 5
- Want to learn absolute positions of all the sensors
  - Applications in indoor positioning
- Image and Video Processing
- Ultrasound Tomography
  - ...

Algorithm

\[
F_E(X, Y) = \|P_E(XY^T - N)\|_F^2 + \lambda \|X\|_F^2 + \lambda \|Y\|_F^2
\]

\[
\hat{\theta}_{j \rightarrow i} = \left( \lambda + \sum_{k \in \partial j \setminus i} \theta_{k \rightarrow j} \theta_{k \rightarrow j}^T \right)^{-1} \left( \sum_{k \in \partial j \setminus i} N_{jk} \theta_{k \rightarrow j} \right)
\]

\[
x_j = \left( \lambda + \sum_{k \in \partial j} \theta_{k \rightarrow j} \theta_{k \rightarrow j}^T \right)^{-1} \left( \sum_{k \in \partial j} N_{kj} \theta_{k \rightarrow j} \right)
\]

Results

Figure 1: Recon. rate vs. \(|E|\)

Figure 2: RMSE vs. iteration for the Netflix Dataset.

Figure 3: MAE vs. iteration for the Movielens Dataset.

Best reported MAE : 0.6382 [2]

References
