Multi-Commodity Flow Models for Dynamic Energy Management

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Smart grid optimization

- increasing need to manage multiple energy commodities coherently over arbitrary network topologies
- models must account for energy conversion, transport and losses, as well as energy storage and dynamic operating conditions (e.g. transmission line failure)
- previous models are either very limited in applicability or not amenable to optimization methods in all but trivial cases
- **we develop a highly-extensible, object-oriented optimization model for multi-commodity energy management that maintains both modeling versatility and efficiently finds globally optimal operation levels**

Flow model formulation

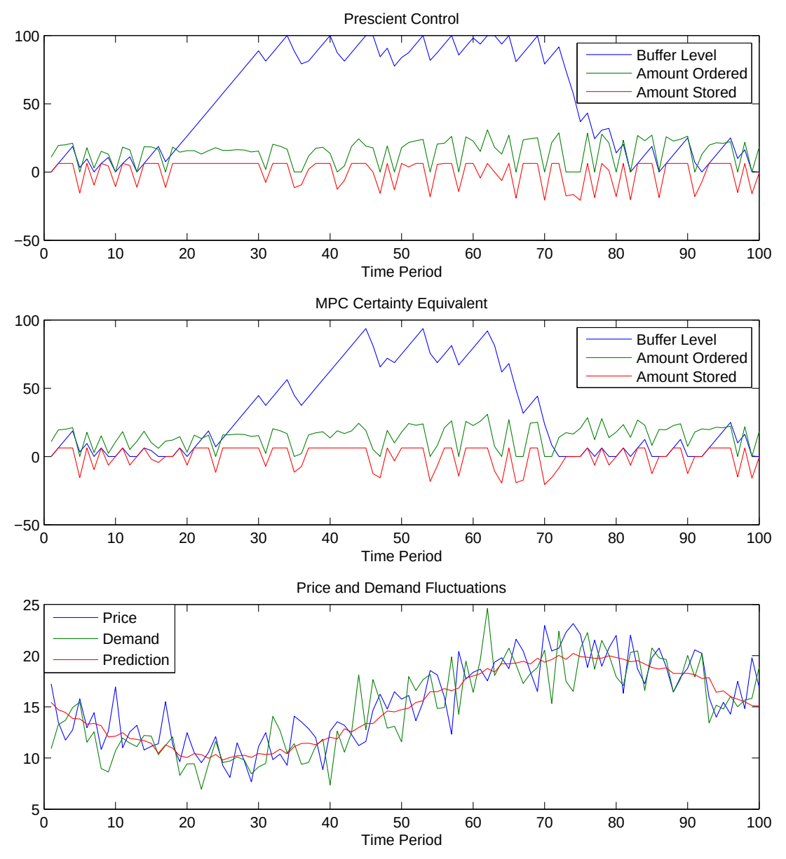
- a *network* consists of a set of *systems* and a set of *edges*
- a system consists of one or more *terminals*, a *constitutive set*, and a real-valued *objective function*.
- each terminal is uniquely associated with a system and consists of an *orientation*, which can be either IN or OUT, an *energy type*, and a real-valued, nonnegative *power*
- the constitutive set of a system is a subset of $\mathbf{R}_+^r$, where r is the number of terminals of the system
- the objective function of a system is a function from the constitutive set into the reals
- an edge connects two terminals of the same energy type and opposite orientations
- the goal of optimal network operation is to minimize the sum of individual system objectives (the *network objective*) subject to terminal powers lying in their respective system's constitutive set

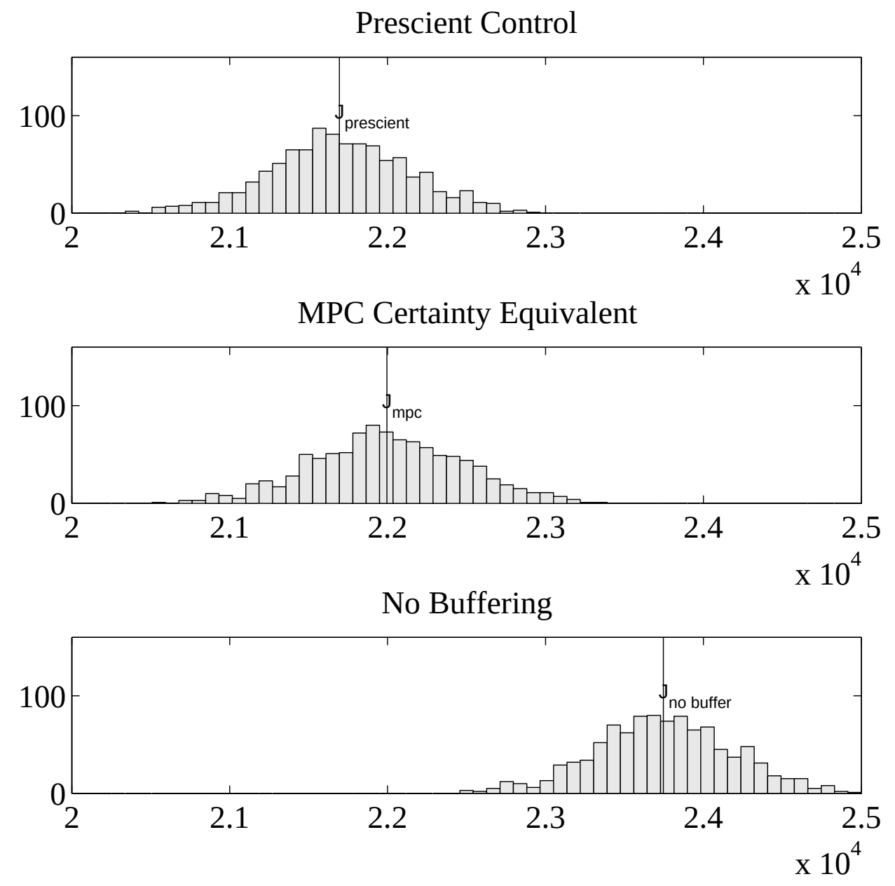
Flow model intuition

- systems are meant to model generators, loads, and other power sources, sinks, and converters
- edges are lossless transporters of power; we can model losses by introducing a system
- terminals are ports on a system through which power flows, either into or out of the system
- constitutive relation of a system tells us what terminal powers are possible for the system and can include dynamic constraints and storage
- objective associated with a system is used to measure the cost (which can be negative, representing revenue) associated with a particular way of operating a system and can include other costs such as CO_2 generation
- **a very wide class of objective functions, conversion efficiency maps, power losses, and generator efficiencies are compatible with convex optimization, and by our formulation so too is the resulting model**
- when this occurs, the problem of operating the network at maximum efficiency can be solved quickly and globally

Example 1: Dynamic buffering

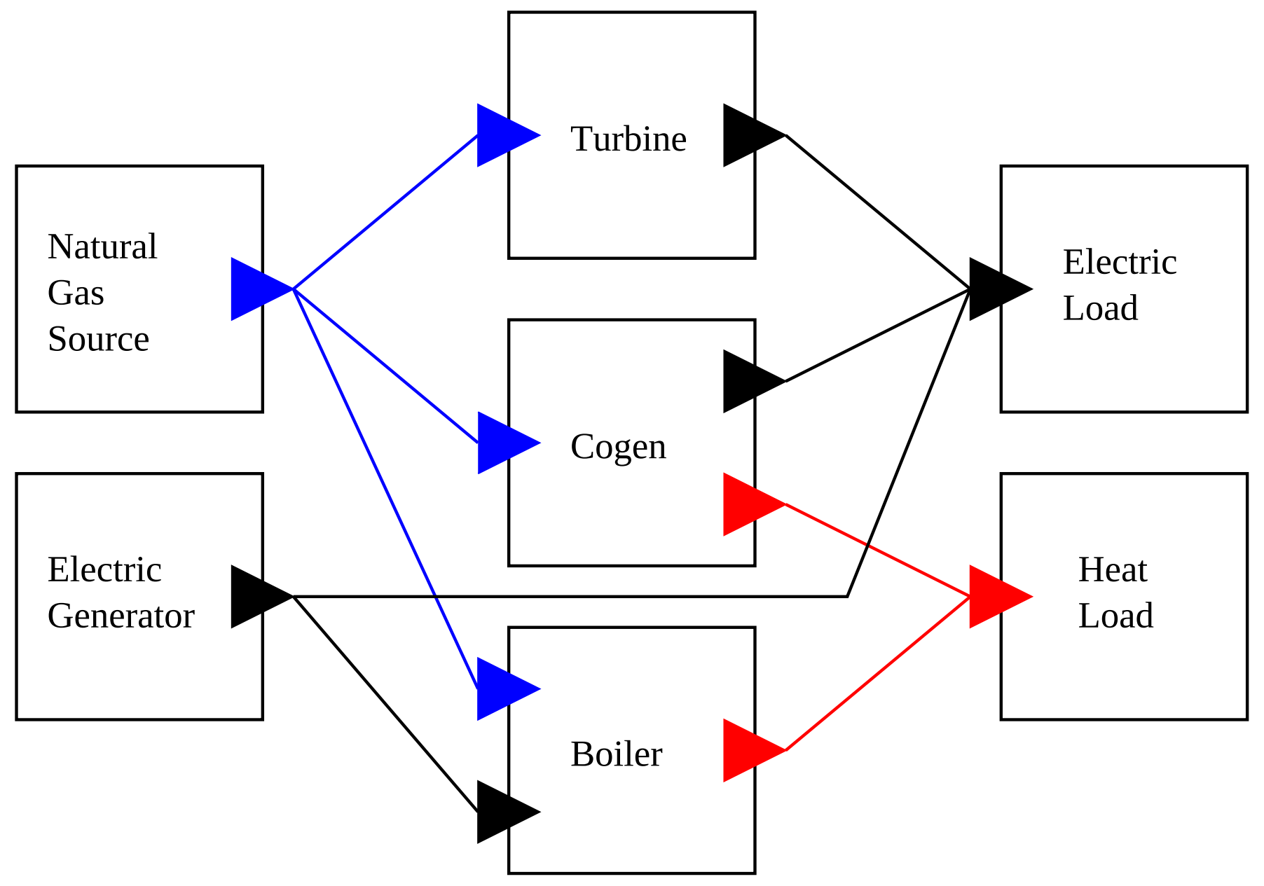
- random prices and demands with predictions
- buffer is capacitated and lossy, and has maximum charge and discharge rates
- goal: minimize energy costs while meeting demand
- presence of buffer allows for price arbitrage through time
- operation of buffer using *model predictive control* (MPC) results in 7.4% savings
- MPC is only 1.2% suboptimal to prescient knowledge of future prices and deman

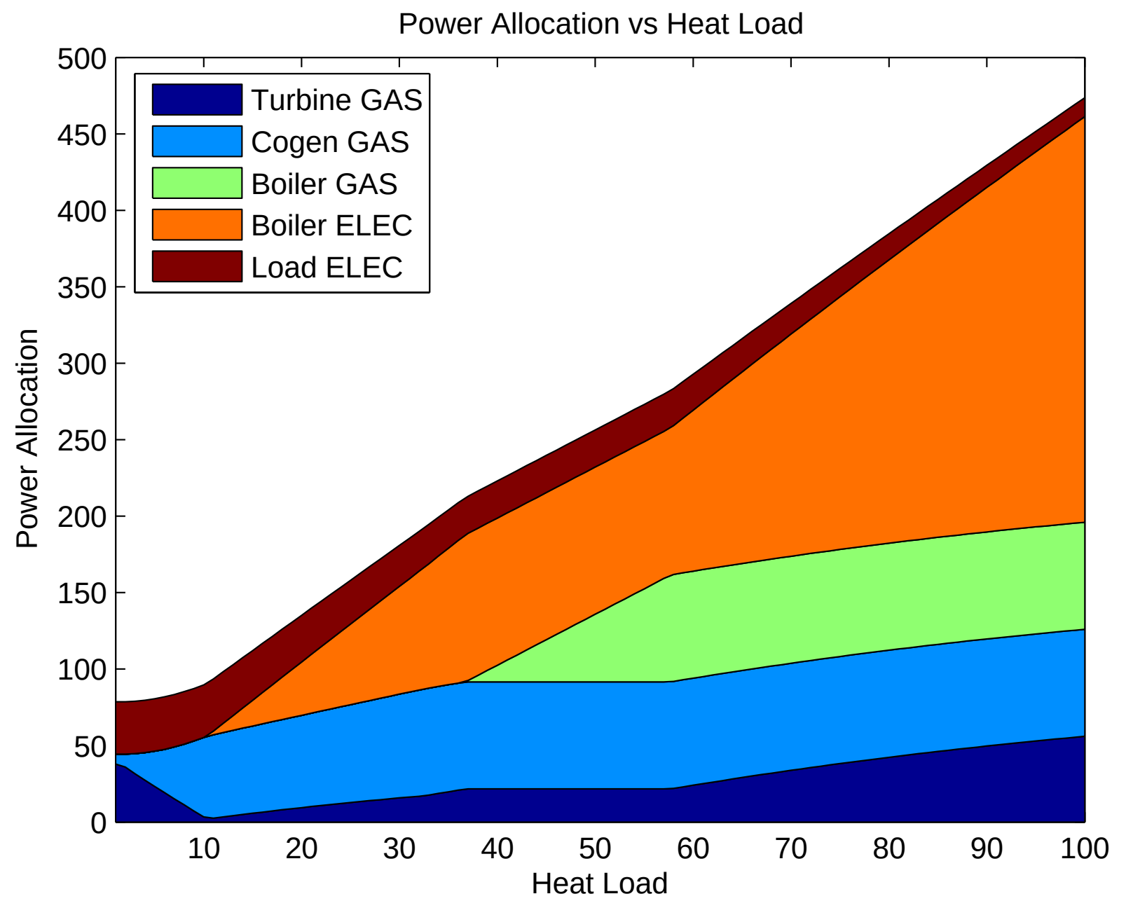


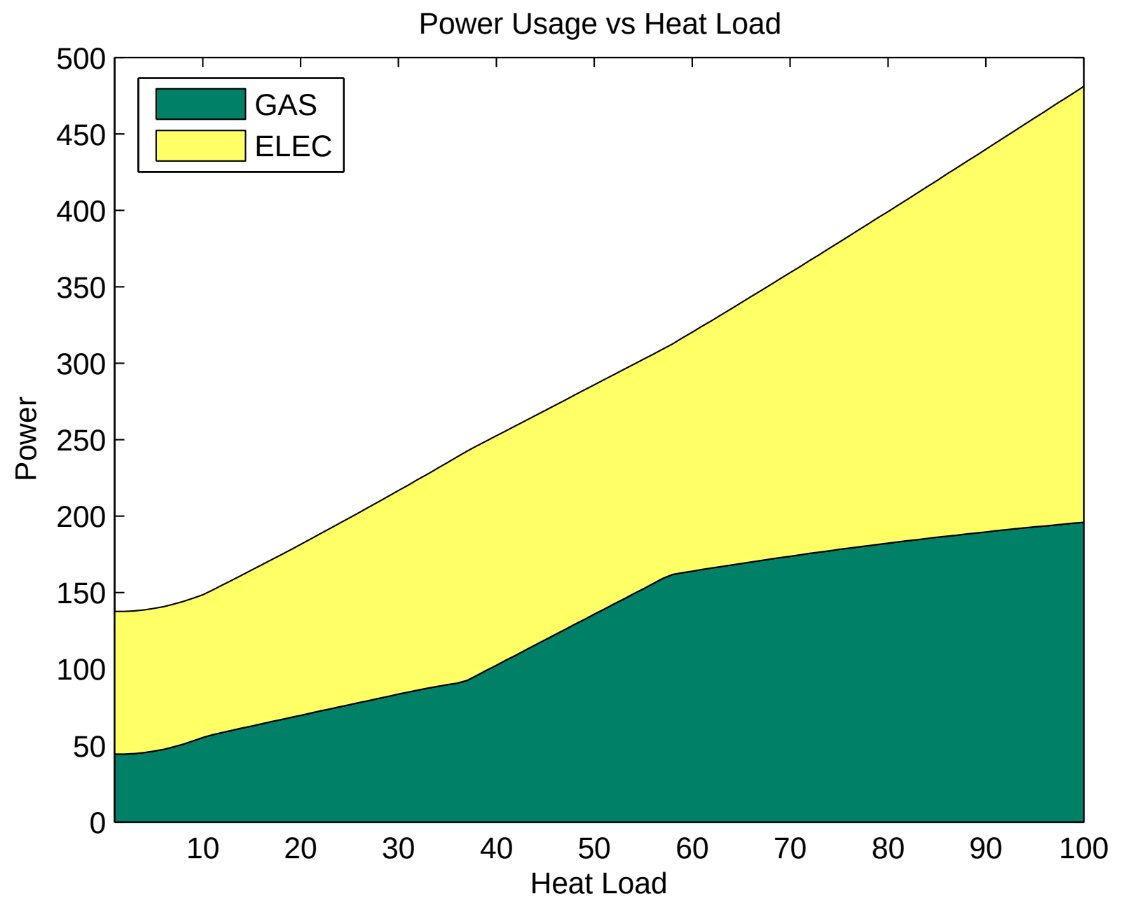


Example 2: A simple multi-commodity network

- a simple energy network with 7 systems, 12 terminals and 9 edges
- blue: natural gas, black: electricity, red: heat
- electric load and heat load must be met by combination of turbine, cogen, generator, and boiler
- all have (nonlinearly varying) efficiencies, capacities
- fixed gas price
- goal: minimize operating cost
- even for such a simple network, results non-obvious
- optimization abstraction allow for simple construction and optimization of large networks without specialized knowledge of optimization algorithms
- model created and solved by very simple code
  - create 8 system objects (7 systems plus 1 network)
    - ∗ e.g. network = System('Network')
  - create 9 edges
    - ∗ e.g. network.connect(gasSource.GAS, turbine.GAS)
  - call network.solve()







Conclusions and Extensions

- flow model allows for rapid prototyping and optimization of both existing and potential energy infrastructures
- maintaining convexity guarantees globally optimal solutions
- simplicity of system drastically expands possible user base
- graphical interface for network construction in develpoment