Optimizing Code Ensembles through Scaling

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Motivation

• Iteratively decoded channel coding (Low complexity)
• Asymptotic analysis (High performance)
• Little known about finite-length performance
  ⇒ Finite-length analysis can lead to better optimization of code ensemble for finite-length irregular LDPC code

Scaling Law

Ballot Problem

• n people vote iid Bernoulli(p), bill is accepted if 2/3 of the people are in favor
• Probability(Bill accepted) = \( Q(Z/\alpha) \)

\[ P_B(n, \lambda, \rho, \epsilon) = Q(\sqrt{n}(\epsilon* - \epsilon - \beta n^{2/3})/\alpha) + O(n^{-1/3}) \]

• \( \epsilon^*, \alpha, \beta \) computed from code ensemble

Low-Density Parity-Check Codes

Variable nodes
Check nodes

Degree Distribution
  • \( \lambda(x), \rho(x) \)
  • Rate = 1 - \( \frac{\rho}{\lambda(x)} \)
Decoder
  • Iterative decoder
  • Message passing

Code Optimization

• Asymptotic analysis: Maximum Rate = 0.7332
• Scaling law: Maximum Rate = 0.7018
  ⇒ Finite-length analysis shows that the cost of having a smaller delay is 4.3% loss in rate