

The background features several large, flowing, abstract shapes in shades of purple, green, and blue. Interspersed among these are numerous small, yellow, triangular shapes that resemble sun rays or confetti, scattered across the white background.

# **Optimization of reservoir management based on approximate dynamic programming**

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# Motivation

- Shortage of **energy** resources calls for better petroleum reservoir management policies
- Optimizing decision-making in reservoir management is challenging
  - **Large-scale nonlinear dynamic** optimization problem
- We propose optimization algorithms based on **Approximate Dynamic Programming (ADP)** to solve this problem

# Introduction to ADP

- Dynamic Programming (DP) potentially achieves global optimum but suffers from the “**curse of dimensionality**”
- ADP tries to keep DP’s merits but overcome the dimensionality curse
- Approach: approximate the *cost-to-go* function as a **linear regression** of a set of **basis functions**:

$$J^*(\mathbf{x}) \approx \tilde{J}(\mathbf{x}) \equiv \sum_{k=0}^K r_k \phi_k(\mathbf{x})$$

# Selection of Basis Functions

Here we apply **Proper Orthogonal Decomposition (POD)**

- Run several simulations with randomized controls, get a set of “snapshots” and then normalize them

$$\bar{\mathbf{x}} = \frac{1}{k} \sum_{i=1}^k \mathbf{x}(i) \quad \mathbf{X} = [\mathbf{x}(1) - \bar{\mathbf{x}}, \mathbf{x}(2) - \bar{\mathbf{x}}, \dots, \mathbf{x}(k) - \bar{\mathbf{x}}]$$

- Apply SVD to get “large-energy” singular vectors

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad \mathbf{\Phi} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N]$$

- Use 1-D polynomials as the basis functions

$$\phi_0(\mathbf{x}) = 1 \quad \phi_1(\mathbf{x}) = \mathbf{u}_1^T \mathbf{x} \quad \phi_2(\mathbf{x}) = (\mathbf{u}_1^T \mathbf{x})^2 \quad \phi_3(\mathbf{x}) = \mathbf{u}_2^T \mathbf{x} \dots$$



# Computation of the Weights

- Various Approximate DP algorithms can be used to compute the weights
  - **Temporal-Difference Learning**
    - Adjusting the weights based on Temporal Difference
  - **Reduced Linear Programming**
    - Theoretically, exact cost-to-go function can be solved by an LP with infinite constraints and variables
    - An approximate solution can be solved by reducing to an LP with finite variables and constraints

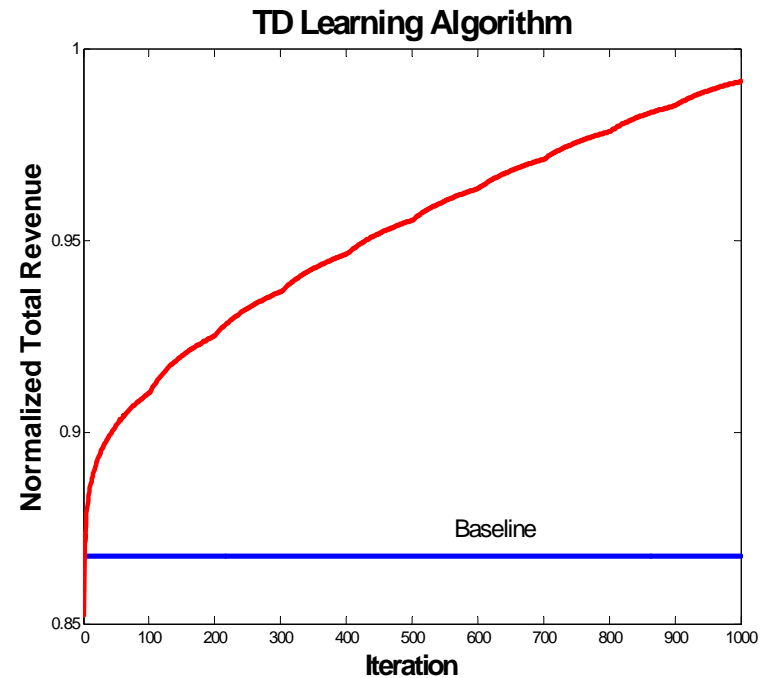
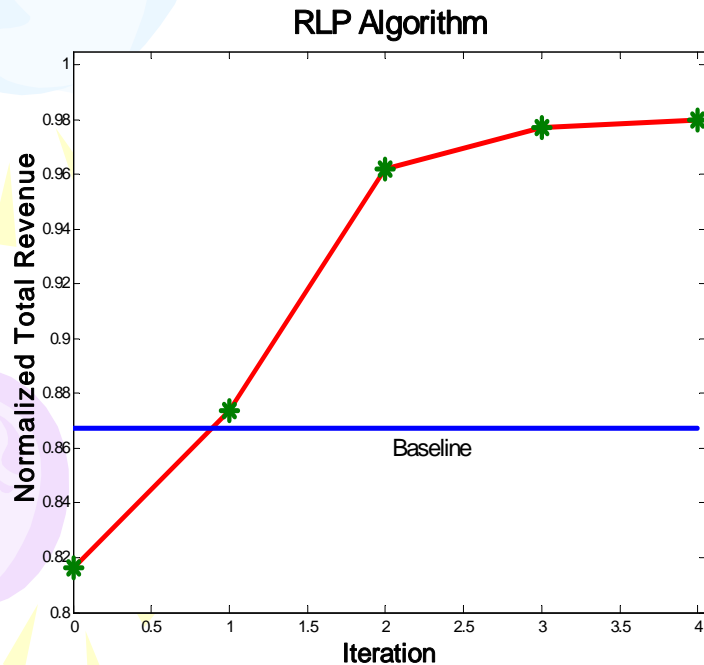


# Our contribution

- Propose a systematic algorithm to choose basis functions based on **POD**
- Extend the classical TD-learning algorithm to “**Full-Simulation**” based TD learning algorithm
- Apply RLP to a **deterministic** optimization system

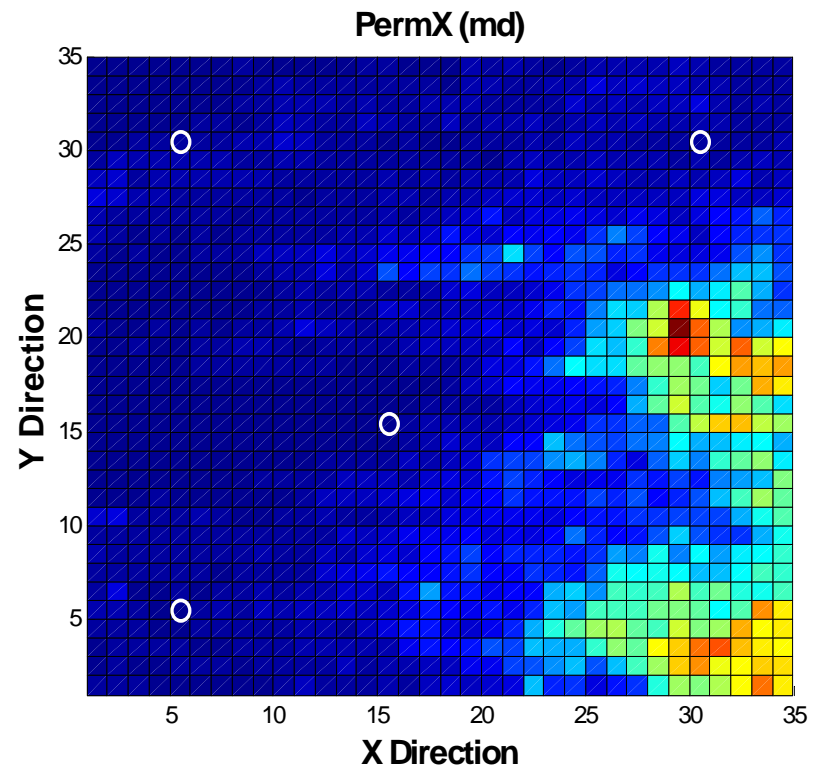
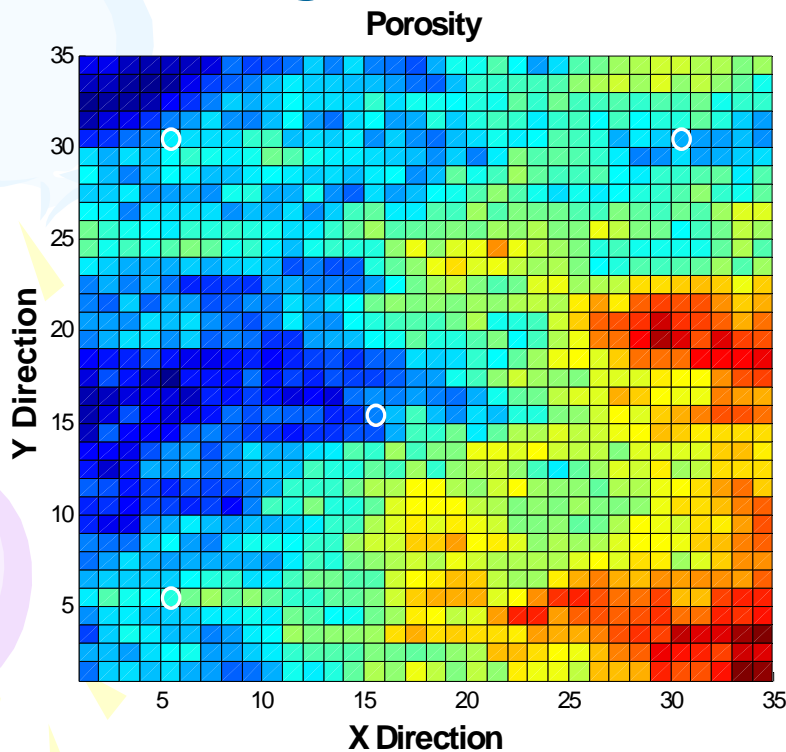
# Validation of ADP

- Primary production in reservoir engineering with special structure is analytically solvable
- Performance of ADP could be verified in this case



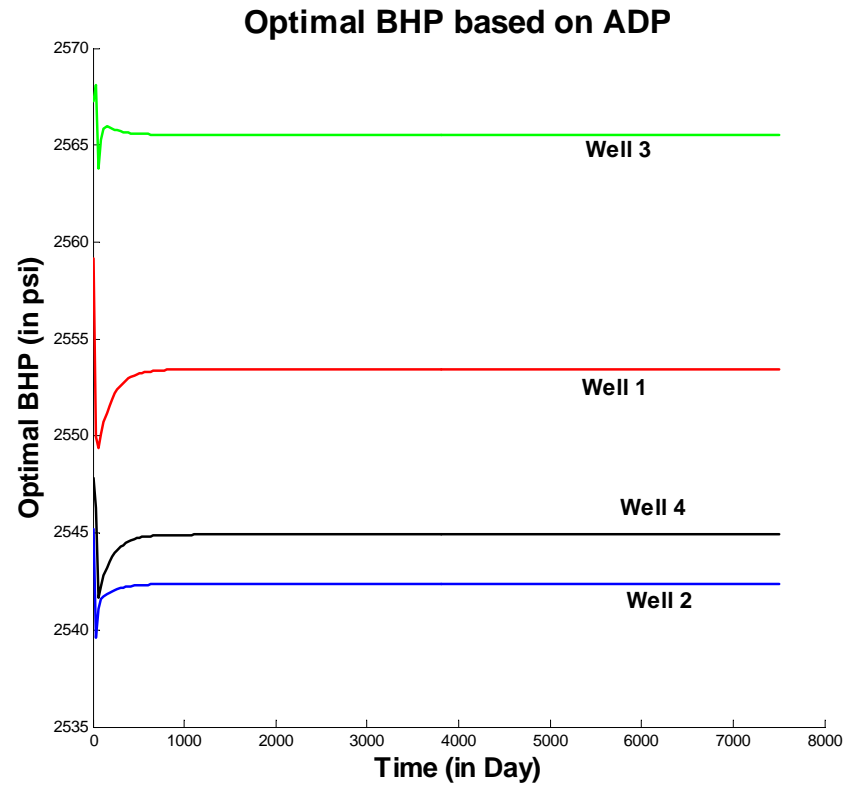
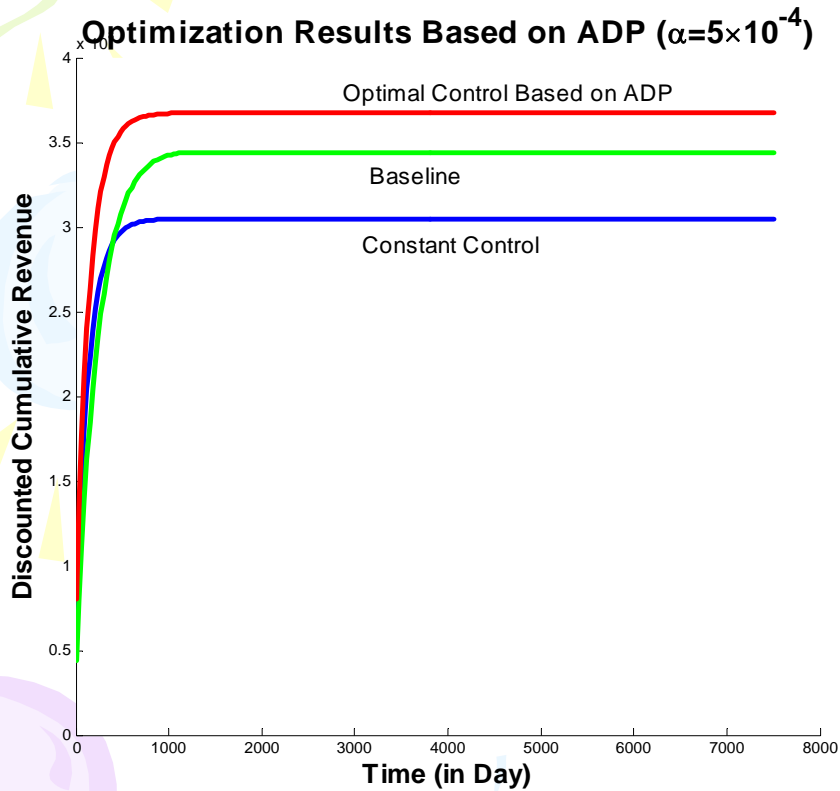
# Application: General Primary Production Problem

- Modified Portion of **SPE-10**
- $35 \times 35 \times 1$  blocks, 4 production wells
- Geological Model is shown below





# Preliminary Results





## Future Work

- Extend the proposed algorithm to some other applications, such as power grid
- Perform some theoretical work to analytically justify the results