Channel Diversity needed for Vector Interference Alignment

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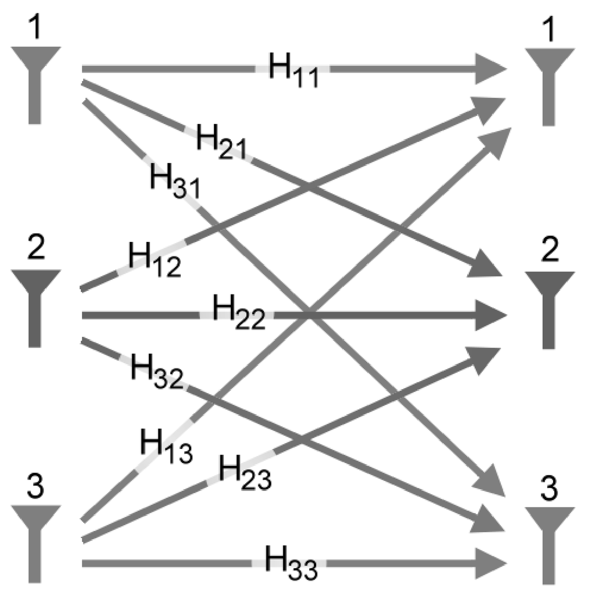
Background

- Interference is the central phenomenon severely limiting the performance of most wireless systems
- Interference alignment has been recently proposed to mitigate interference
  - Design signals such that, upon arriving at unintended receivers, they overlap with each other and the resulting interference is perceived as much less than the sum of the individual interferences
- Interference alignment achieves K/2 degrees of freedom
  - At high-SNR, each user can communicate as if it has half the resources of the channel for its exclusive use, independent of the number of users
- However, the construction requires the channel diversity, i.e., the number of independent channel realizations (e.g. the time needed, or the bandwidth), to grow exponentially in the number of users
- Our goal is to study the trade-off between DoF and channel diversity

Channel Model

- Consider K-user fully-connected real-valued fast-fading interference channel
  - Each receiver wants to obtain a message from its corresponding transmitter, but the signal is superimposed by interferences from other transmitters



$$\mathbf{y}_i = \sum_{j=1}^{K} \mathbf{H}_{ij}\mathbf{x}_j + \mathbf{z}_i$$

$$\mathbf{H}_{ij} = \begin{bmatrix} h_{ij}^{(1)} & & \\ & \ddots & \\ & & h_{ij}^{(L)} \end{bmatrix}$$

Interference Alignment

- Vector space interference alignment strategies [1]:
  - Message vector $\hat{\mathbf{x}}_i \in \mathbb{R}^D$, apply linear precoding $\mathbf{x}_i = \mathbf{V}_i\hat{\mathbf{x}}_i$ to map $\hat{\mathbf{x}}_i$ to a subspace V_i , receiver decodes by zero forcing interference
  - Decoding is successful if signal subspace is disjoint from interference subspace

$$\mathbf{H}_{ii}V_i \cap \left(\sum_{j\neq i} \mathbf{H}_{ij}V_j\right) = \{0\}$$

$$\mathrm{DoF} = \max_{\{V_i\}\text{ satisfies decoding condition }\forall i} KD/L$$

Trade-off between DoF and Diversity

- For 3-user interference channel [2],

$$\mathrm{DoF} = \frac{3}{2}\left(1 - \frac{1}{4L - 2\lfloor L/2 \rfloor - 1}\right)$$

The gap between the DoF and K/2 decreases as 1/L

- We show that for K ≥ 4 users,

$$\mathrm{DoF} \leq \frac{K}{2}\left(1 - \frac{1}{11\sqrt{L}}\right)$$

The gap between DoF and K/2 decreases at most as $1/\sqrt{L}$

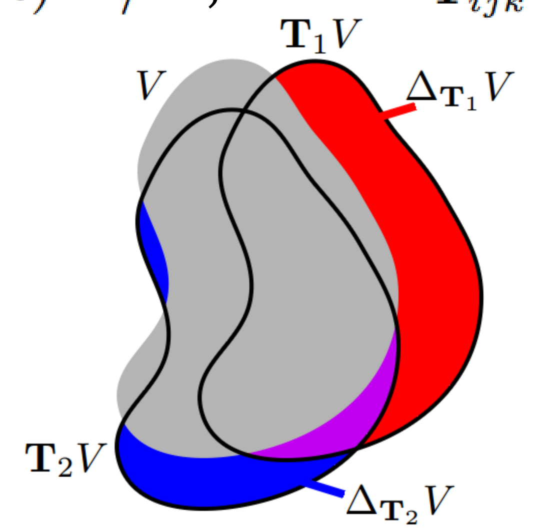
- Moreover, if K is larger than the order of $\sqrt{\log L}$, then the gap decreases at most as $1/\sqrt[4]{L}$

Alignment Width

- Define the alignment width of a subspace as

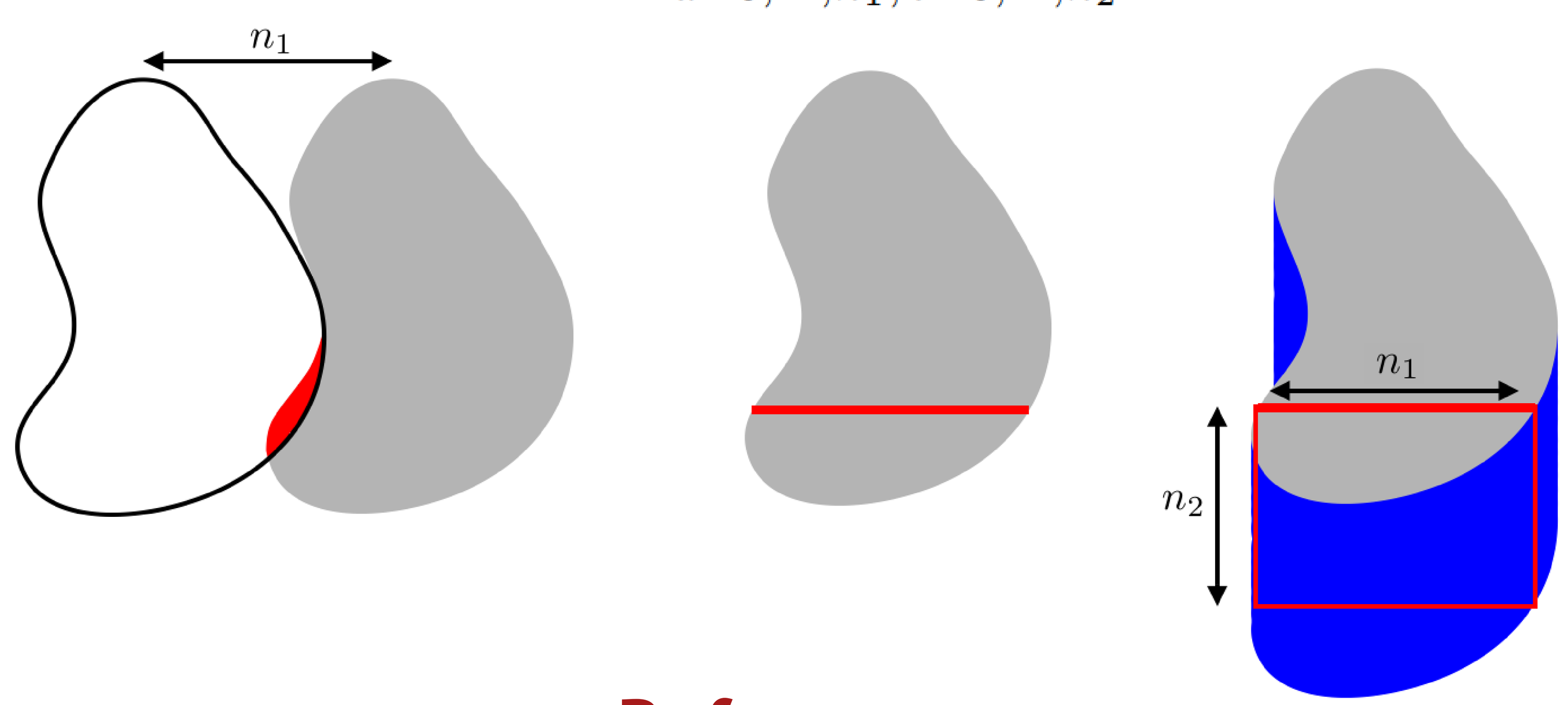
$$\begin{aligned}\Delta_{\mathbf{T}}V &= \dim V - \dim(V \cap \mathbf{T}V) \\ &= \dim(V + \mathbf{T}V) - \dim V\end{aligned}$$

- The subspaces V_i need to align to the transformations $\mathbf{T}_{ijk} = \mathbf{H}_{1i}^{-1}\mathbf{H}_{1k}\mathbf{H}_{jk}^{-1}\mathbf{H}_{ji}$
  - If $\mathrm{DoF} \geq (1-\epsilon)K/2$, then $\Delta_{\mathbf{T}_{ijk}}V_i \leq 2\epsilon L$



Intuition of the Proof

- Compared to V , the dimension of $V \cap \mathbf{T}_1V$ is reduced by $\Delta_{\mathbf{T}_1}V$
- As long as $n_1\Delta_{\mathbf{T}_1}V < D$, $V \cap \cdots \cap \mathbf{T}_1^{n_1}V \neq \{0\}$
- We can find $\mathbf{v}$ such that $\mathrm{span}\{\mathbf{v}, \mathbf{T}_1\mathbf{v}, ..., \mathbf{T}_1^{n_1}\mathbf{v}\} \subseteq V$
- As long as $n_2\Delta_{\mathbf{T}_2}V < L - D$, $V + \cdots + \mathbf{T}_2^{n_2}V \neq \mathbb{R}^L$
- Hence $\mathrm{span}\{\mathbf{T}_1^a\mathbf{T}_2^b\mathbf{v}\}_{a=0,\ldots,n_1,\, b=0,\ldots,n_2} \neq \mathbb{R}^L$,$n_1n_2 \leq L$



References

[1] V. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," IEEE Trans. Inf. Theory, vol. 54, no. 8, pp. 3425-3441, Aug. 2008.

[2] G. Bresler and D. Tse, "3 user interference channel: Degrees of freedom as a function of channel diversity," in Communication, Control, and Computing, 2009. Allerton 2009. 47th Annual Allerton Conference on, 2009, pp. 265-271.