Communication with Limited Feedback

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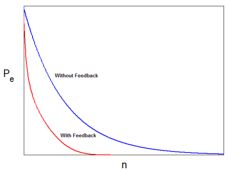
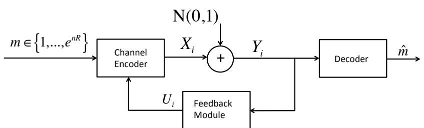
Status Quo

- The feedback does not increase the capacity for memory-less channels

  for e.g. Gaussian: $C_{FB}=\frac{1}{2}\ln(1+P)$
- Can reduce the complexity of encoder/ decoder
- Can increase the decay rate of error probability as function of blocklength
- Can reduce energy per bit

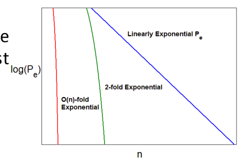


Gaussian Channel with Feedback



- [Shannon 59]: With no feedback, the error probability decays at most linearly exponential with blocklength

$$\Pr\{\hat{m}\neq m\}\geq e^{-O(n)}$$



- [Schalkwijk and Kailath 66]: With perfect (noiseless and unlimited) feedback, for $R<C$, a simple linear scheme can achieve

$$\Pr\{\hat{m}\neq m\}\leq e^{-e^{O(n)}}$$

- [Gallager and Nakiboglu 08]: With perfect feedback, best achievable error is

$$\Pr\{\hat{m}\neq m\}\leq \exp(-\underbrace{\exp(\ldots\exp(O(n))\ldots)}_{O(n)})$$

- [Kim, Lapidoth and Weissman 09]: With noisy feedback the error exponent is finite

$$\Pr\{\hat{m}\neq m\}\approx e^{-O(n)}$$

- [Polyaskiy, Poor and Verdu 10]: Noiseless feedback reduces the minimum energy per bit when nR is fixed and $n\rightarrow\infty$.

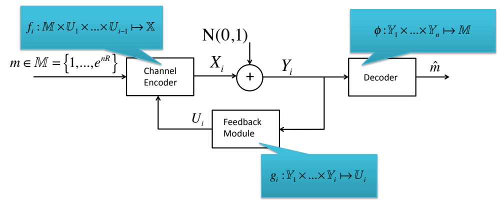
References

[1] Reza Mirghaderi, Andrea Goldsmith and Tsachy Weissman, "**Achievable Error Exponents for Gaussian Channels with Rate-Limited Feedback**", submitted to IEEE Trans. Information Theory, arXiv:1007.1986v1

[2] Reza Mirghaderi and Andrea Goldsmith, "**Communication with Rate-Limited Feedback**", Allerton Conference on Communications, Control and Computing 2010

[3] Reza Mirghaderi and Andrea Goldsmith, "**Energy-Efficient Communication via Feedback**", presented in CISS 2012, Princeton, NJ

AWGN Channel with Rate-Limited Feedback



- Constraints

$$E\left[\sum_{i=1}^{n}|X_i|^2\right]\leq nP$$

$$|U_1|\times\ldots\times|U_n|\leq e^{nR_{FB}}$$

- Objective:

  Choose $\{U_i\},\{g_i\},\{f_i\}$ and ϕ to maximize the decay rate of error probability

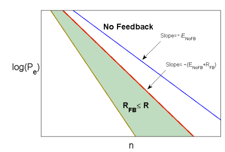
$$P_e(n,R,R_{FB},P)$$

Main Results

A super-exponential error probability is achievable if and only if $R_{FB}\geq R$

- $R_{FB}<R$: The error exponent is finite but higher than no-feedback error exponent

$$P_e(n,R,R_{FB},P)\leq e^{-n(E_{NoFB}(R)+R_{FB}+o(1))}$$

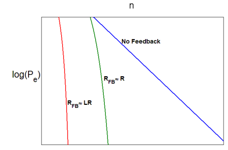


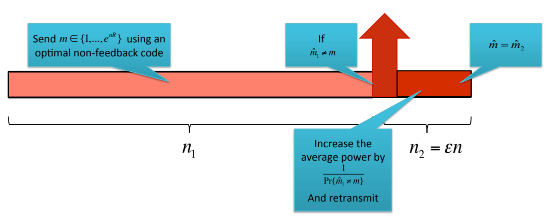
- $R_{FB}\geq R$: Double exponential error probability

$$P_e(n,R,R_{FB},P)\leq e^{-e^{O(n)}}$$

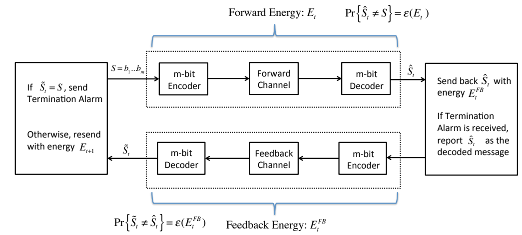
- $R_{FB}\geq LR$: L-fold exponential error probability

$$P_e(n,R,R_{FB},P)\leq \exp(-\underbrace{\exp(\ldots\exp(O(n))\ldots)}_{L})$$





Feedback under Delay/Energy Constraint



- Constraints

  Decoding Delay $\leq T$

$$\sum_{t=1}^{T}(E_t+E_t^{FB})\leq E_{tot}$$

- Objective:

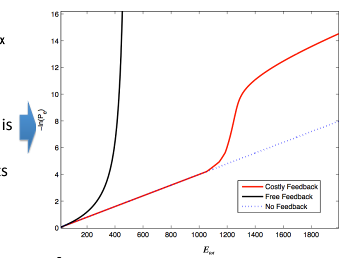
  Choose $\{E_t,E_t^{FB}\}_{t=1}^{T}$ to minimize the overall probability of error : $P_e(E_{tot},T)$

Main Results

Feedback gain highly depends on the error probability model $\varepsilon(.)$

Exponential Error Model: $\varepsilon(x)=\beta e^{-\alpha x}$

- Suits scenarios where Tx energy dominates
- Feedback gain is high if total energy is large enough
- **No feedback gain** for energy budgets **below** a threshold



Super-Exponential Error Model: $\varepsilon(x)=\beta e^{-\alpha x^2}$

- Suits some scenarios where Tx and coding energy are comparable
- **No feedback gain** for energy budgets **above** a threshold

Future Work

- Constructing simple capacity achieving codes using rate-limited feedback
- Characterize the energy constrained feedback communication under a broader class of energy consumption models
- Designing feedback schemes for point-to-point links with asymmetric forward and feedback energy costs

Feedback to improve performance can be highly effective or strictly sub-optimal