Efficient Computation of Shapley Values for Demand Response Programs

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Summary

- The Shapley Value is a concept that can be used to generate a fair payout scheme. We apply this concept to Demand Response (DR) programs and we propose an efficient method for estimating the Shapley Value using stratified sampling.

Motivation

- Many DR programs can be formulated in a game theory setting as **coalition games**.
- The Shapley Value concept provides a **fair payment incentive** that guarantees every participant is better off staying in the DR program.
- Exact computation of the Shapley Value is a **combinatorial calculation** and so estimation techniques are important. A modest DR program with 500 participants would require over 10^{150} calculations per participant to calculate the exact value.

The Shapley Value

- Given a supermodular **payment function** f which calculates the payment returned to any specified coalition (subset of participants).
- The Shapley Value for a given participant x is a **weighted average** of the marginal contributions of that participant to all possible coalitions.

Marginal Contribution:

$$\rho_X(\mathcal{Y}) = f(\mathcal{Y} \cup \{x\}) - f(\mathcal{Y})$$

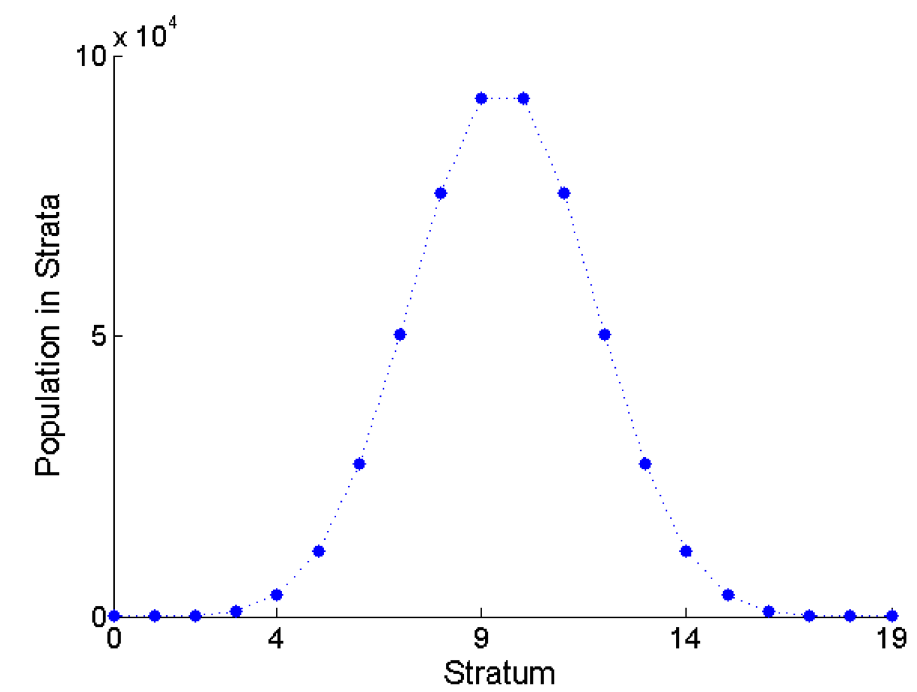
Shapley Value:

$$\phi_x(f) = \sum_{\mathcal{A} \subseteq \mathcal{X} \setminus \{x\}} \frac{|\mathcal{A}|!(|\mathcal{X}| - |\mathcal{A}| - 1)!}{|\mathcal{X}|!} \rho_x(\mathcal{A})$$

- Aside from directly computing the above formula, **generating functions** are also used to calculate the Shapley Value. This approach requires large arrays to reduce complexity to polynomial time.
- Monte-Carlo techniques are also used by **randomly sampling coalitions.**

Stratified Sampling

- A method of sampling by dividing the population into **strata** and sampling from each strata.
- The **population** here is the marginal contributions of a given participant to every possible coalition.
- We divide the population into strata where each strata contains marginal contributions corresponding to a fixed subset size



Comparing Sampling Methods

- It can be shown that the Shapley Value is equal to the mean of the average marginal contribution from each strata.
- By sampling appropriately from the stratum, the variance of the Shapley Value estimate can be reduced.

Random Sample
Sample randomly, disregarding the strata

$$\sigma^2_{RS} = \frac{1}{N}\left[\mathbb{E}[\boldsymbol{\sigma}^2] + \mathrm{Var}(\boldsymbol{\mu})\right]$$

Equal Bin Sample
Sample an equal number of times from each strata

$$\sigma^2_{EB} = \frac{1}{N}\mathbb{E}[\boldsymbol{\sigma}^2]$$

Standard Deviation Sample
Sample from each strata in proportion to the standard deviation of the population segment in the strata

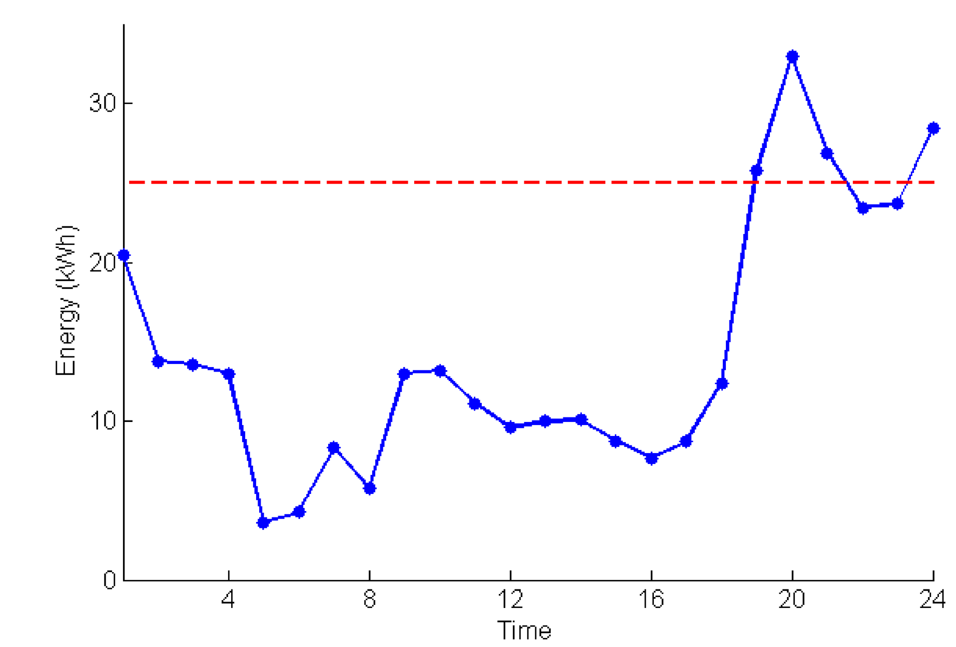
$$\sigma^2_{SD} = \frac{1}{N}\mathbb{E}[\boldsymbol{\sigma}]^2$$

$$\sigma^2_{SD} \leq \sigma^2_{EB} \leq \sigma^2_{RS}$$

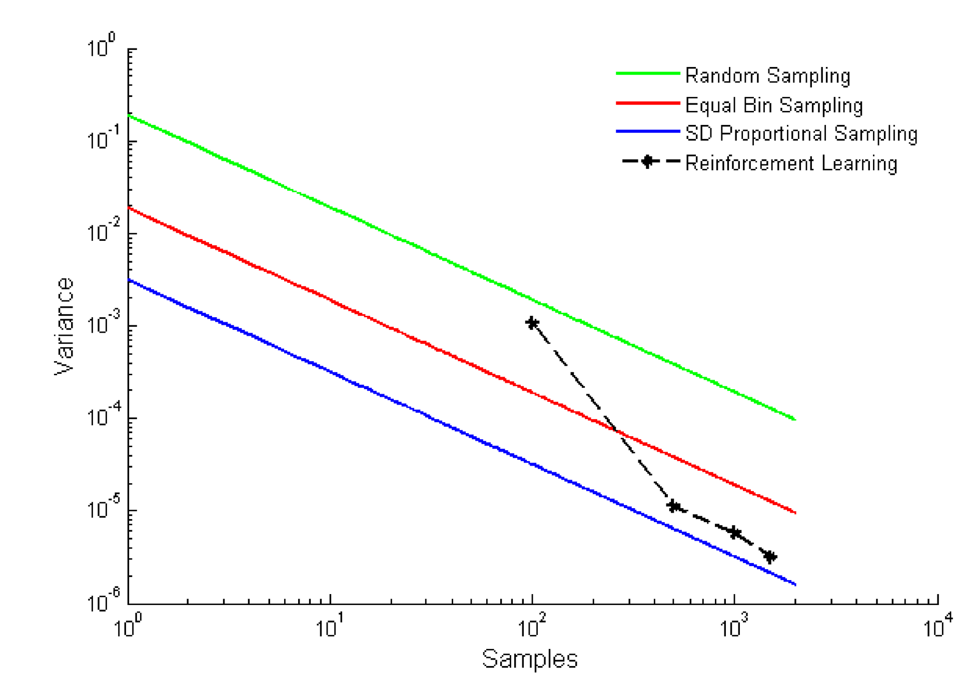
Results for Load Shedding DR Program

- A limit is placed on the aggregate load. For every time step during which aggregate load exceeds this limit, all loads that consumed power during this time will be penalized in proportion to their Shapley Value.

$$f(\mathcal{X}) = \left[\sum_{x \in \mathcal{X}} x - M\right]_+$$



- As the number of samples we take increases, the variance of the Shapley Value estimate decreases.
- A reinforcement learning algorithm that approximates SD Sampling was also implemented.



procedure STANDARDDEVIATIONSAMPLING(N, ϵ_t)
  $t \leftarrow 1$
  $S_i = \{\emptyset\}, \quad \forall i$
  while $t \leq N$ **do**
    Choosen bin i at random and weighted by $\pi_i(t)$
    $s \leftarrow \rho_\alpha(x, i)$ ▷ sample from bin i
    $S_i \leftarrow \{S_i \cup s\}$ ▷ add sample to set
    $\hat{\sigma}_i \leftarrow \sqrt{\mathbb{E}[S_i^2] - (\mathbb{E}[S_i])^2}$ ▷ update SD
    $t \leftarrow t + 1$
  end while
  $\hat{\phi}_\alpha(f) \leftarrow \mathbb{E}[\,\mathbb{E}[S_i] \mid i\,]$
  return $\hat{\phi}_\alpha(f)$
end procedure

▷ Note that in this algorithm, we take the convention that if a bin contains ≤ 1 samples the variance is 0.