Improved Capacity Approximations

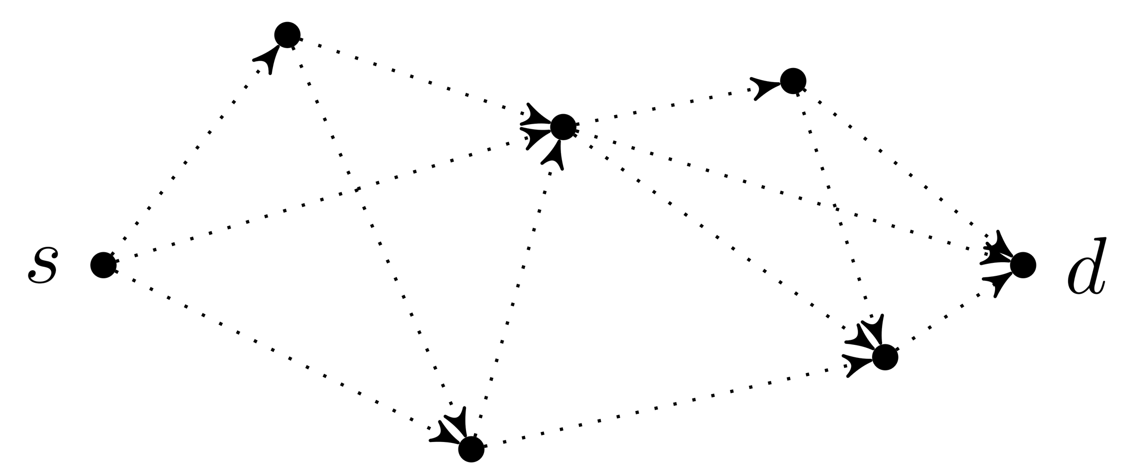
Ritesh Kolte, Bobbie Chern, Ayfer Özgür
{rkolte, bgchern, aozgur}@stanford.edu

Center for Science of Information
NSF Science and Technology Center

Introduction

- Recently there has been significant progress in characterizing the capacity regions of Gaussian relay networks
- Schemes such as Quantize-Map-Forward (QMF) and Noisy Network Coding achieve rates with bounded gap to capacity, independent of SNR and channel gains
- In this scheme, relays quantize their received signal at the noise level and map into a random transmission codebook
- However, the gap is **linear in the number of nodes**

Noisy Network Coding



Noisy Network Coding Achievable Rate

$$R \geq \max_{\prod_{k\in\mathcal{N}} p(x_k)p(\hat{y}_k|y_k,x_k)} \min_{\Omega\subseteq\mathcal{N}} I(X_\Omega;\hat{Y}_{\Omega^c}|X_{\Omega^c}) - I(Y_\Omega;\hat{Y}_\Omega|X_\mathcal{N},\hat{Y}_{\Omega^c})$$

Cutset Upper Bound

$$R \leq \max_{p(x_\mathcal{N})} \min_{\Omega\subseteq\mathcal{N}} I(X_\Omega;Y_{\Omega^c}|X_{\Omega^c})$$

The following three factors contribute to the gap between the achievable rate and the cutset bound:

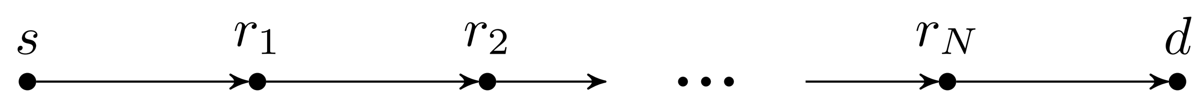
1. Beamforming Loss: Maximization over product distributions
2. Quantization Noise introduced at the relays: $I(X_\Omega;\hat{Y}_{\Omega^c}|X_{\Omega^c})$ instead of $I(X_\Omega;Y_{\Omega^c}|X_{\Omega^c})$
3. Rate Penalty for communicating quantization indices to the destination: $I(Y_\Omega;\hat{Y}_\Omega|X_\mathcal{N},\hat{Y}_{\Omega^c})$

There is a tradeoff between the 2nd and 3rd factors: finer quantizations reduce the quantization noise introduced at the relays but increase the rate penalty for communicating the quantization indices.

Closer examination reveals that **coarser quantization exploits this tradeoff better than finer quantization**, as illustrated in the following simple example.

Simple Example: Line Network

At every node, there is a transmit power constraint P and the additive noise in the received signal is i.i.d. $\mathcal{CN}(0,1)$. For simplicity, assume unit gain links.



Though it is trivial to obtain the capacity $\log(1+P)$ of the line network via decode-forward, we use it to illustrate our key idea which is then applied to configurations where decode-forward is far from optimal.

The quantization at the ith relay node is chosen independently as $\hat{Y}_i = Y_i + \hat{Z}_i$ where $\hat{Z}_i \sim \mathcal{CN}(0,Q)$. In existing literature, quantization at relays is done at the noise level i.e. $Q \sim 1$.

$$I(X_\Omega;\hat{Y}_{\Omega^c}|X_{\Omega^c}) = \log\left(1+\frac{P}{Q+1}\right) \geq \log(1+P) - \log(1+Q)$$

$$I(Y_\Omega;\hat{Y}_\Omega|X_\mathcal{N},\hat{Y}_{\Omega^c}) = (|\Omega|-1)\log\left(1+\frac{1}{Q}\right) \leq \frac{N}{Q}$$

$$Q=1:\ \text{gap} \leq \log 2 + N$$
$$Q=N:\ \text{gap} \leq \log(N+1) + 1$$

General Gaussian Networks

For general Gaussian networks, when the quantization noise variance is chosen to be equal to Q, a bound on the gap to capacity can be obtained as follows:

$$\text{gap} \leq d_0^* \log\left(1+\frac{N}{d_0^*}\right) + \frac{N}{Q} + d_Q^* \log(Q+1),$$

where

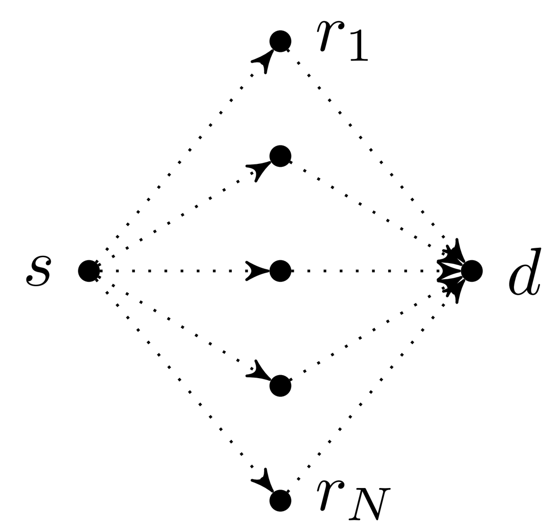
$$d_Q^* = \text{DOF}\left(\arg\min_\Omega I(X_\Omega;\hat{Y}_{\Omega^c}|X_{\Omega^c})\Big|_Q\right)$$
$$= \text{DOF}\left(\arg\min_\Omega \log\det\left(I+\frac{P}{(Q+1)\sigma^2}H_{\Omega\to\Omega^c}H^\dagger_{\Omega\to\Omega^c}\right)\right)$$

This expression can be used to guide the choice of quantization so that the gap to capacity is minimzed.

It can also be used to obtain explicit expressions for an improved gap to capacity in many important special cases such as:

- Diamond Network
- Layered Network: Fast Fading Links
- Layered Network: Arbitrary Phase Gain Links

Gaussian Diamond Network

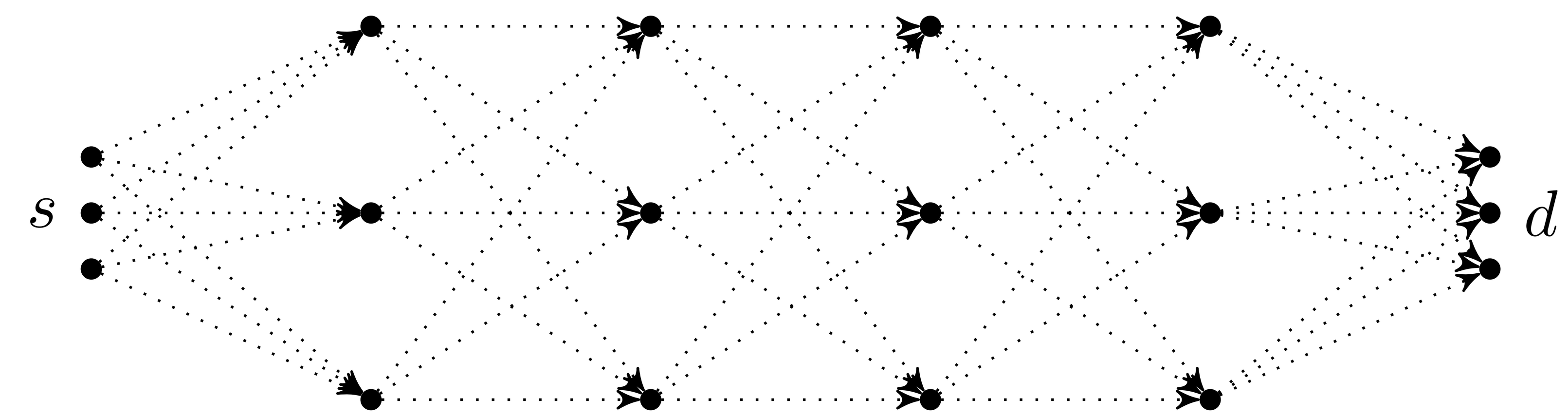


Arbitrary Gains

Improved gap obtained by choosing **quantization noise variance proportional to** N

Old gap $\leq 1.26N$
New gap $\leq 2\log(N+1)+1$

Gaussian Layered Networks



D layers, K nodes per layer

Improved gaps obtained by choosing **quantization noise variance proportional to** D

- Fast Fading Links

Old gap $\leq 1.26KD$
New gap $\leq K\log D + K$

- Arbitrary Phase Gain Links

Old gap $\leq 1.26KD$
New gap $\leq K^2\log D + 3K\log K$

References

1 Bobbie Chern, Ayfer Özgür, "Achieving the Capacity of the N-Relay Gaussian Diamond Network Within log N Bits", IEEE Information Theory Workshop 2012 Lausanne

2 Ritesh Kolte, Ayfer Özgür, "Improved Capacity Approximations for Gaussian Relay Networks", IEEE Information Theory Workshop 2013 Seville