Level Crossing Statistics of Multi-hop Fading Channels in Amplify-and-Forward Relay Networks

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Background and Motivation

How often does a random process cross a certain threshold?

- Rice's formula [1] for the Level Crossing Rate (LCR)

$$LCR_r(u)=\int_0^\infty \dot{r}\cdot f_{r=u}(r,\dot{r})\,d\dot{r}$$

✖ Depends on the joint statistics of the process and its time derivative
✖ Becomes infinite for some correlation models
✖ How to deal with inherently discrete random processes?

- Another look at the problem: Level crossings of sampled random processes [2]

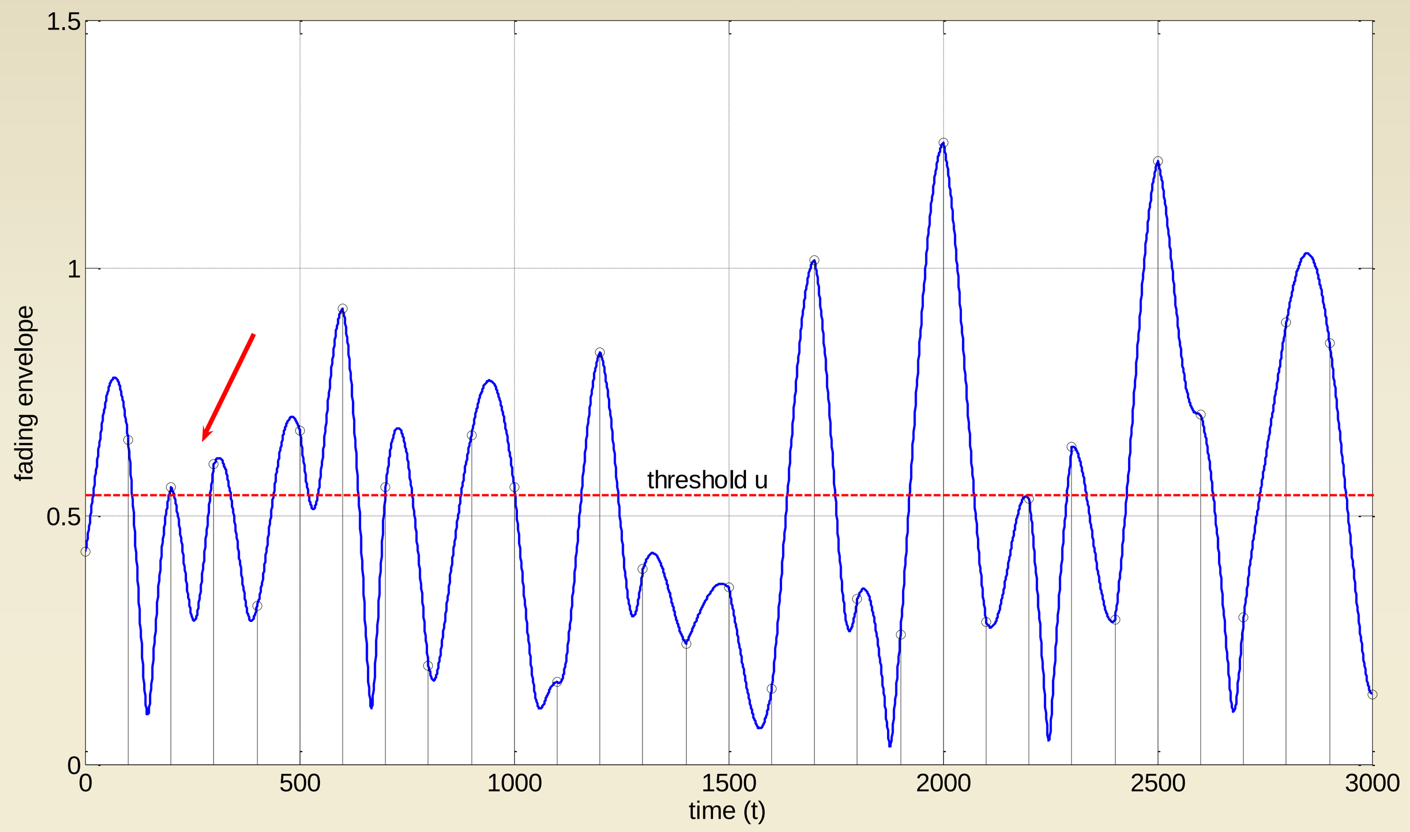

Fig. 1. Level crossings in continuous and sampled random processes

$$\left.\begin{array}{l} R_1=R(t)\\ R_2=R(t+T_s)\end{array}\right\} LCR_r(u;T_s)=\frac{1}{T_s}\Pr\{R_1<u,R_2>u\}_{\rho(T_s)}$$

- LCR can be expressed in terms of the 1-D and 2-D CDF:

$$LCR_r(u;T_s)=\frac{F_r(u)-F_{r,r}(u,u;\rho(T_s))}{T_s}$$

✓ Depends on the joint statistics of the process and its time correlation
✓ Finite LCR for finite sampling period Ts
✓ LCR of sampled random process upper-bounded by Rice's LCR
✓ The average fade duration (AFD) can be also calculated as

$$AFD_r(u;T_s)=\frac{F_r(u)}{LCR_r(u;T_s)}$$

- **Motivations:**
  ✓Can we use our approach to characterize the LCR of other random processes of interest?
  ✓Can we obtain results where Rice's approach fails to do so?
  ✓Case of study:

  ➔ **Amplify-and-Forward Multi-hop Relay Networks**

  ✓ What is the impact of the number of hops in the LCR/AFD?
  ✓ What is the impact of the relay mobility?

Multi-hop Relay Networks (I)

Multi-hop communications:

- Amplify-and-forward (AF) fixed-gain (FG) multi-hop relay networks.
- Semi-blind relays: Only statistical knowledge of CSI.

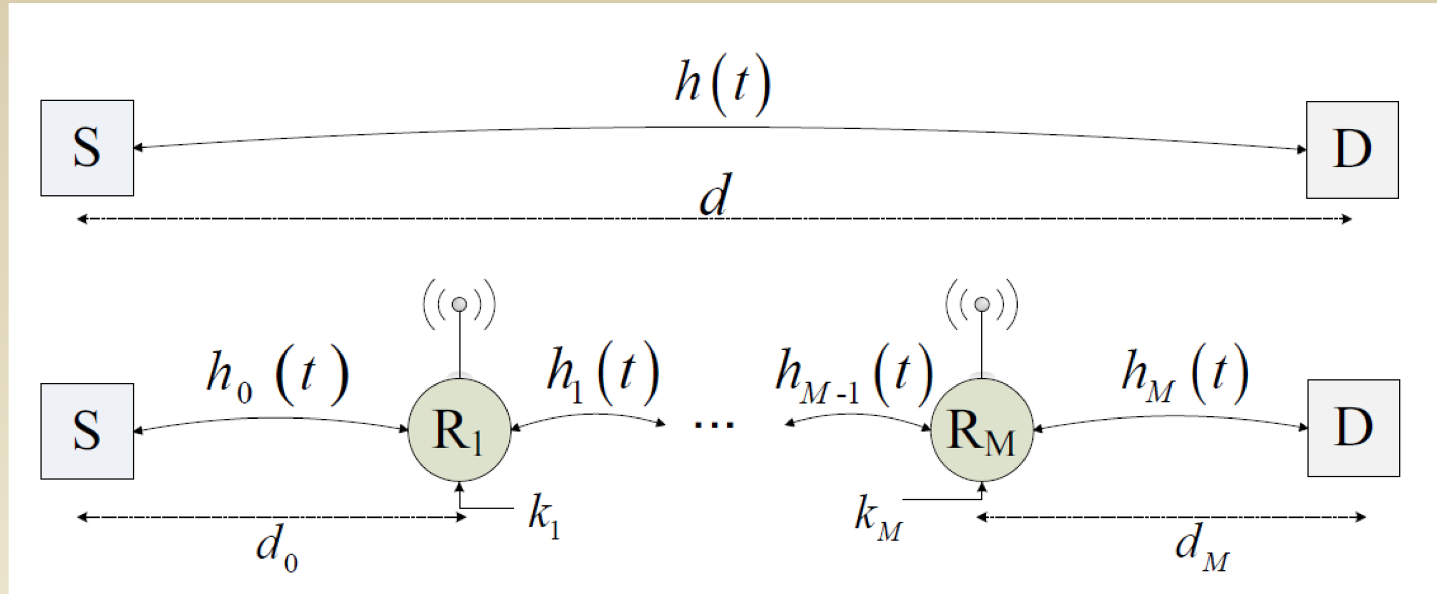

Fig. 2. Communication schemes, no relay vs. multi-hop relay network

LCR for the equivalent channel gain (cascaded fading channel)
✓ Only approximate results for Rayleigh fading in the literature
✓ Product of random variables: only first-order characterization (PDF/CDF)

$$h_{eq}(t)=\prod_{i=0}^{M}k_i h_i(t)$$

- **Our approach [3]:**
  ✓Log-normal (LN) distribution
  - Product of independent LN is LN ➔ Analytical tractability
  ✓ Continuous LCR using [1], discrete LCR using [2]

$$N_H(v)=\frac{\sqrt{\ddot{\rho}}}{2\pi}e^{-\frac{[20\log_{10}v-\mu_x]^2}{2\Omega_x}}.$$

$$N_H(v,T_S)=\frac{1}{\pi T_S}\sqrt{\frac{1-\rho_{T_S}}{1+\rho_{T_S}}}e^{-\frac{(20\log_{10}(v)-\mu_x)^2}{2\Omega_x}}\Phi_1\left(\tfrac{1}{2},1,\tfrac{3}{2};-\tfrac{1-\rho_{T_S}}{1+\rho_{T_S}};-\tfrac{1}{2\Omega_x}\tfrac{1-\rho_{T_S}}{1+\rho_{T_S}}\left[20\log_{10}(v)-\mu_x\right]^2\right),$$

- Discrete LCR converges to continuous as Ts ➔ 0
- Discrete LCR admits exponential correlation model (continuous LCR is ∞)

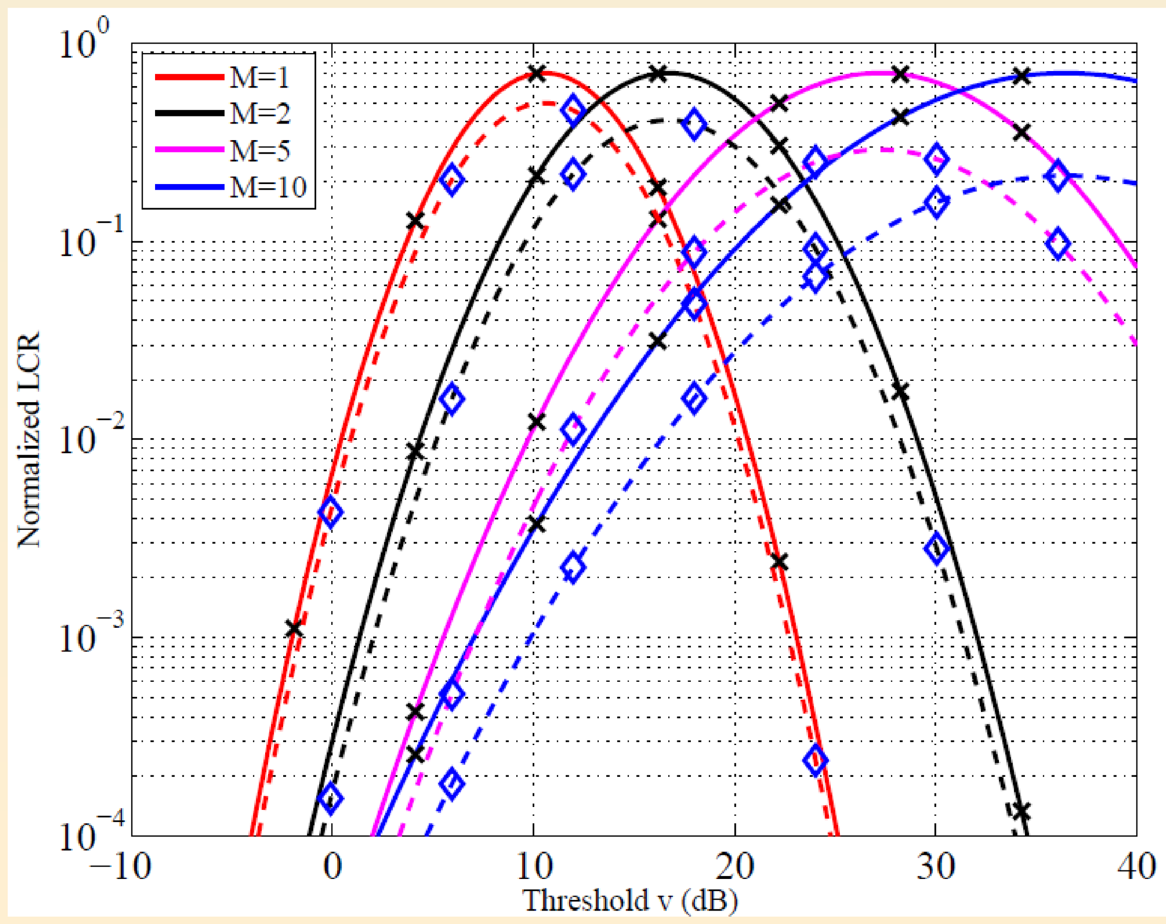

Fig. 3. LCR of multi-hop channel gain for different correlation profiles.

Multi-hop Relay Networks (II)

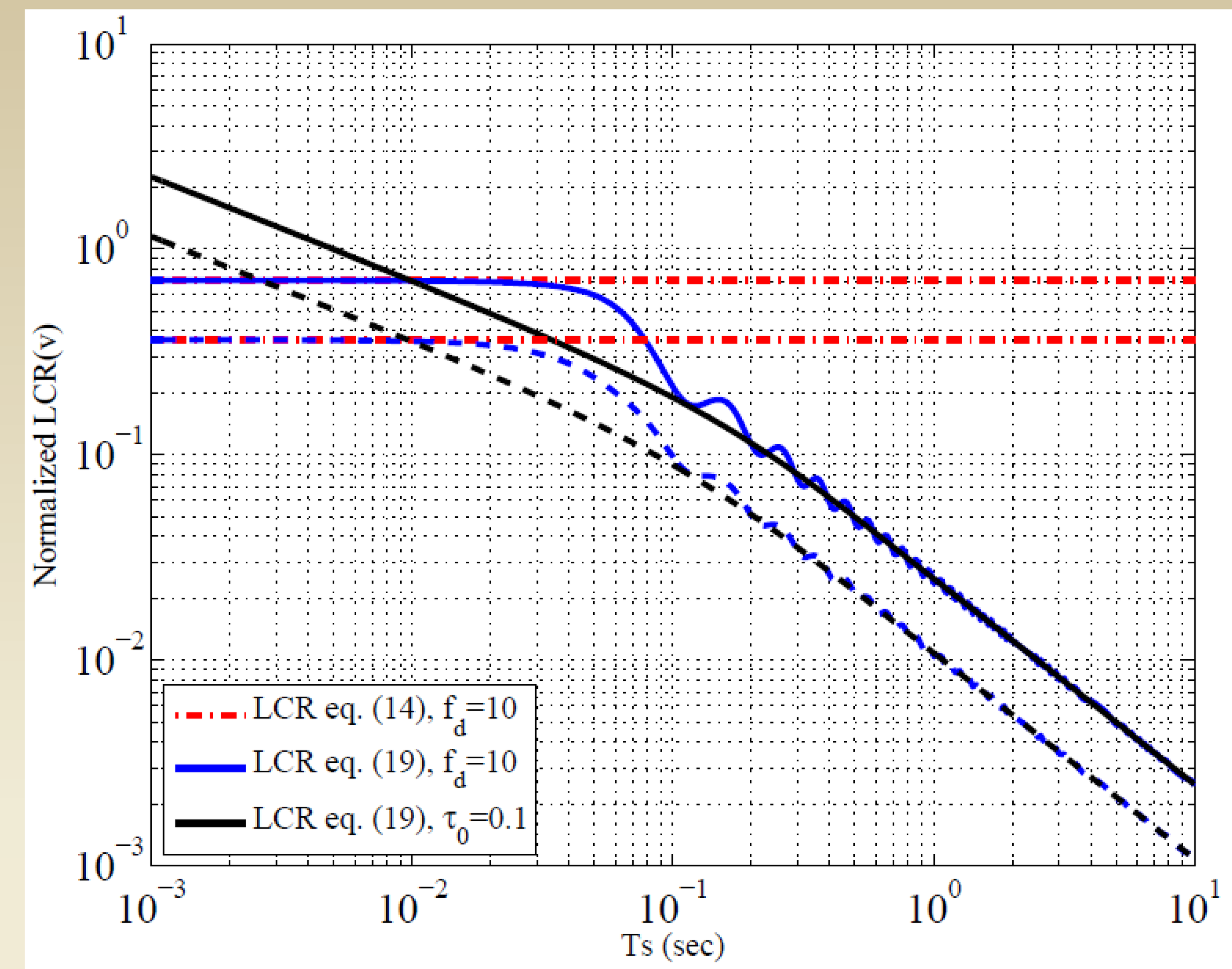

Fig. 4. Continuous vs. discrete LCR as a function of Ts

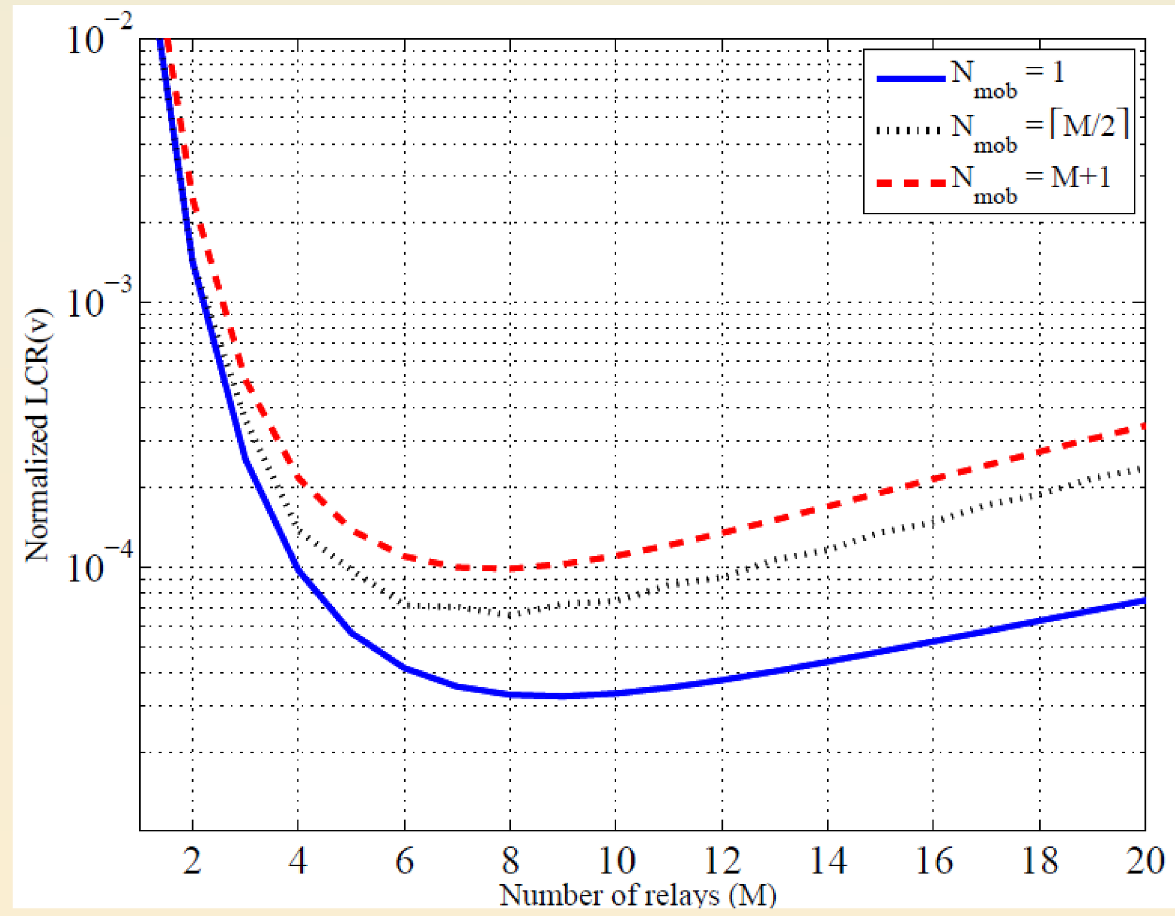

Fig. 5. Effect of relay mobility in the number of crossings as a function of M.

- **Insights**
  ✓Effect of M in the number of crossings ➔ feedback design
  ✓Effect of relay mobility ➔ Is it worth using this relay/should we skip it?
  ✓Effect of finite Ts ➔ estimation of number of crossings

- **Future work:**
  - Product of correlated LN is LN ➔ Correlation between hops
  - Product of positive RVs is approximated by LN ➔ Other distributions

References

[1] S. Rice, "Mathematical Analysis of Random Noise". Bell Telephone Laboratories, 1944.
[2] F. J. Lopez-Martinez, E. Martos-Naya, J. F. Paris, and U. Fernandez-Plazaola, "Higher Order Statistics of Sampled Fading Channels with Applications," IEEE Trans. Veh. Technol., vol. 61, no. 7, pp. 3342–3346, 2012.
[3] F. J. Lopez-Martinez, E. Kurniawan, A. Goldsmith, "Average Fade Duration for Amplify-and-Forward Relay Networks in Log-Normal Fading" Submitted to IEEE Globecom 2013.

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