**INTRODUCTION**

In many future wireless networks, we will encounter sensor nodes that harvest the energy they need for communication from natural resources in their environment.

**GOAL**

Understand the capacity of such newly emerging communication systems and the optimal principles to design and operate them. Provide insights on the following foremost engineering questions:

- How does the capacity of an energy-harvesting AWGN channel depend on major system parameters such as the battery size and the energy harvesting process? Are there different operating regimes where this dependence is qualitatively different?
- For a system powered with a certain energy harvesting mechanism, can we provide a rule of thumb for choosing the battery size?
- What are the properties of the energy harvesting process that are critical to capacity in the finite battery regime? Consequently, what are more desirable and less desirable energy-harvesting profiles?

**SYSTEM MODEL**

<table>
<thead>
<tr>
<th>Battery</th>
<th>( B_{\text{max}} )</th>
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<tbody>
<tr>
<td>( E_t )</td>
<td>( X_t )</td>
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Battery Constraint and Update:

\[
|X_t|^2 \leq B_t \quad B_{t+1} = \min(B_t + E_{t+1} - |X_t|^2, B_{\text{max}})
\]

Bernoulli Energy Arrival Process:

\[
E_t = \begin{cases} E & \text{w.p. } p \\ 0 & \text{w.p. } 1 - p \end{cases}
\]

\( E_t \) are i.i.d. Bernoulli random variables. The inter-arrival time \( T_k \) between non-zero energy packets will be a Geometric(\( p \)) random variable. The energy arrivals are causally known only at the transmitter.

**A NEAR OPTIMAL ONLINE POLICY**

Goal: Maximize the Long Term Average Rate:

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} \log (1 + \tilde{g}(B_t, t))
\]

The maximization is over \( \mathcal{G} \), the class of feasible online policy. Using Law of Large Number, and properties of the Geometric r.v., it’s possible to turn the original problem into the following more tractable convex optimization problem:

\[
\max \left\{ \tilde{g}(j) \right\}_{j=0}^{\infty} \sum_{j=0}^{\infty} (1 - p)^j \frac{1}{2} \log(1 + \tilde{g}(j))
\]

subject to:

\[
\sum_{j=0}^{\infty} \tilde{g}(j) \leq B_{\text{max}} \quad \tilde{g}(j) \geq 0 \quad \text{for } \forall j
\]

**THEOREM 1**

Let \( \tilde{g}(j) \) be a maximizer to the above optimization problem, and let \( \tilde{g}(j) = (1 - p)^j B_{\text{max}} \), be a feasible policy to the problem. Then for \( \forall p \in (0,1) \) and \( \forall B_{\text{max}} \in (0, \infty) \):

\[
\sum_{j=0}^{\infty} (1 - p)^j \frac{1}{2} \log(1 + \tilde{g}(j)) - \sum_{j=0}^{\infty} (1 - p)^j \frac{1}{2} \log(1 + \tilde{g}(j)) \leq 0.973
\]

**EXTENSIONS TO OTHER ENERGY PROFILES**

Extend our constant gap upper and lower bound approximation of capacity to more general energy harvesting profiles:

- \((k\text{-level Distribution})\) Each \( E_t \) is i.i.d with probability \( p_i \) being some positive number \( E_t \), for \( i = 1, 2, \ldots, k \), and \( 1 - \sum_{i=1}^{k} p_i \) being 0. For any fixed \( k \), we have the gap no more than \( \frac{1}{k} \log(k) + 2.58 \).
- \((\text{Uniform Distribution})\) Each \( E_t \) has i.i.d. uniform \([E_1, E_2] \) distribution. We have a gap of no more than 3.08 bits.

However, counter-examples exist. Understand what conditions on the distribution are needed and how we can improve our achievable strategy to get a constant gap will be our future work.

**REFERENCES**