Near Optimal Energy Control and Approximate Capacity of Energy Harvesting Communication

Yishun Dong, Farzan Farnia, Ayfer Özgür
{ydong2, farnia, aozgur}@stanford.edu

Center for Science of Information
NSF Science and Technology Center

Introduction

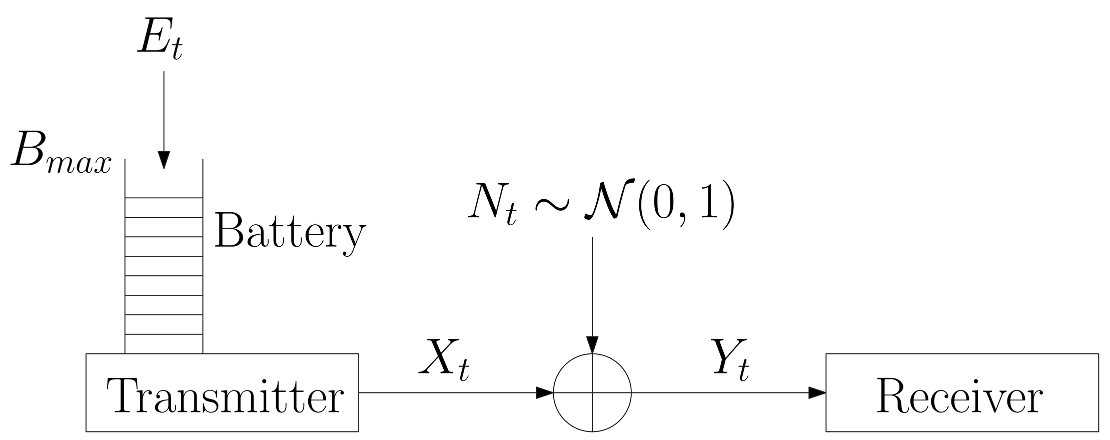
In many future wireless networks, we will encounter sensor nodes that harvest the energy they need for communication from natural resources in their environment.

Goal

Understand the capacity of such newly emerging communication systems and the optimal principles to design and operate them. Provide insights on the following foremost engineering questions:

- How does the capacity of an energy-harvesting AWGN channel depend on major system parameters such as the battery size and the energy harvesting process? Are there different operating regimes where this dependence is qualitatively different?
- For a system powered with a certain energy harvesting mechanism, can we provide a rule of thumb for choosing the battery size?
- What are the properties of the energy harvesting process that are critical to capacity in the finite battery regime? Consequently, what are more desirable and less desirable energy-harvesting profiles?

System Model



Battery Constraint and Update:

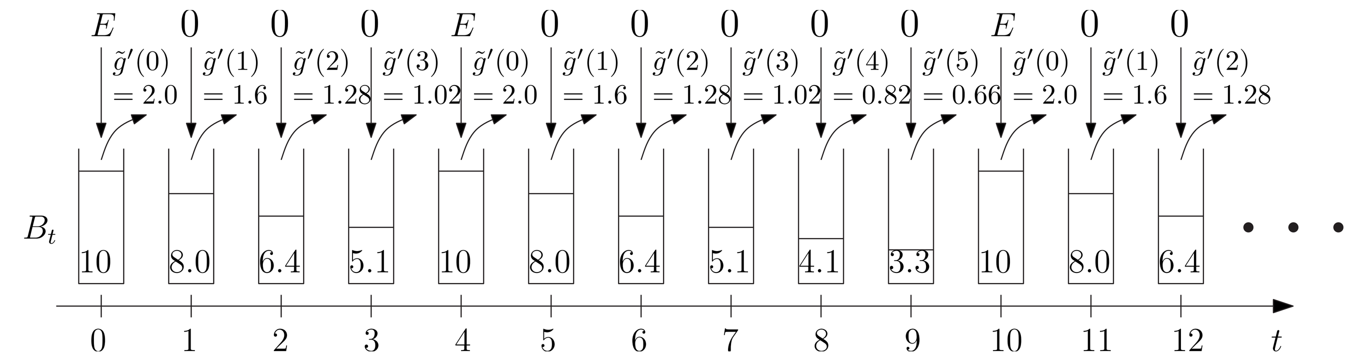
$$\begin{aligned} |X_t|^2 &\le B_t \\ B_{t+1} &= \min(B_t + E_{t+1} - |X_t|^2, B_{max}) \end{aligned}$$

Bernoulli Energy Arrival Process:

$$E_t = \begin{cases} E & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$$

E_t are i.i.d. Bernoulli random variables. The inter-arrival time T_k between non-zero energy packets will be a Geometric(p) random variable. The energy arrivals are causally known only at the transmitter.

A Near Optimal Online Policy



Goal: Maximize the Long Term Average Rate:

$$\max_{g\in\mathcal{G}} \lim_{N\to\infty} \inf \frac{1}{N}\sum_{t=1}^{N} \frac{1}{2}\log\left(1+g(B_t,t)\right)$$

The maximization is over $\mathcal{G}$, the class of feasible online policy. Using Law of Large Number, and properties of the Geometric r.v., it's possible to turn the original problem into the following more tractable convex optimization problem:

$$\begin{aligned} \max_{\{\tilde{g}(j)\}_{j=0}^{\infty}} \quad & \sum_{j=0}^{\infty} p(1-p)^j \tfrac{1}{2}\log(1+\tilde{g}(j)) \\ \text{subject to:} \quad & \sum_{j=0}^{\infty} \tilde{g}(j) \le B_{max} \\ & \tilde{g}(j) \ge 0 \quad \text{for } \forall j \end{aligned}$$

Theorem 1: Let $\{\tilde{g}^*(j)\}_{j=0}^{\infty}$ be a maximizer to the above optimization problem, and let $\tilde{g}'(j) = p(1-p)^j B_{max}$, be a feasible policy to the problem. Then for $\forall p \in (0,1)$ and $\forall B_{max} \in (0,\infty)$:

$$\sum_{j=0}^{\infty} p(1-p)^j \frac{1}{2}\log(1+\tilde{g}^*(j)) - \sum_{j=0}^{\infty} p(1-p)^j \frac{1}{2}\log(1+\tilde{g}'(j)) \le 0.973$$

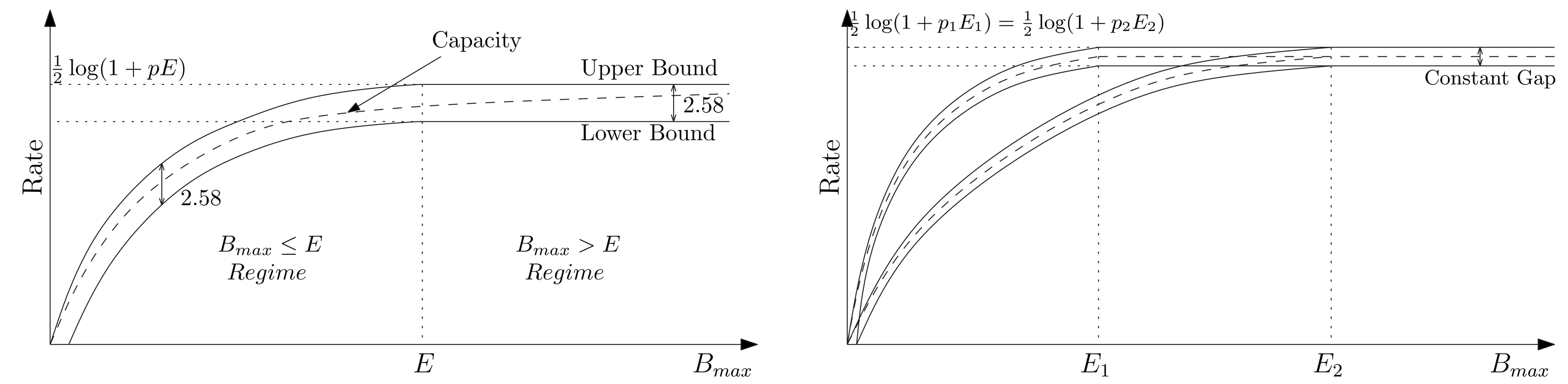
Extensions to Other Energy Profiles

Extend our constant gap upper and lower bound approximation of capacity to more general energy harvesting profiles:

- (k-level Distribution) Each E_t is i.i.d with probability p_i being some positive number E_i for $i = 1, 2, ..., k$, and $1-\sum_{i=1}^{k} p_i$ being 0. For any fixed k, we have the gap no more than $\frac{1}{2}\log(k)+2.58$.
- (Uniform Distribution) Each E_t has i.i.d Uniform$[E_1, E_2]$ distribution. We have a gap of no more than 3.08 bits.

However, counter-examples exist. Understand what conditions on the distribution are needed and how we can improve our achievable strategy to get a constant gap will be our future work.

Constant Gap on Capacity



Regime I: $B_{max} \le E$

Upper Bound based on $B_{max} = \infty$ Capacity:

$$C \le \frac{1}{2}\log\left(1+pB_{max}\right) \triangleq C_{ub}(B_{max},p)$$

Achievable Scheme with Receiver Side Information:

$$R_{ach} \ge \sum_{j=0}^{\infty} p\,(1-p)^j \max_{p(x):|X|^2\le\mathcal{E}_j} I(X;Y)$$

- The amplitude constrained mutual information maximization is no more than 1.04 bits lower than the classical AWGN capacity.
- The capacity improvement due to the receiver side information can not be more than $H(p)$ bits.

$$C \ge \sum_{j=0}^{\infty} p(1-p)^j \frac{1}{2}\log\left(1+\tilde{g}'(j)\right) - H(p) - 1.04 \triangleq C_{lb}(B_{max},p)$$

Theorem 2: For $\forall\ 0<p<1$, $B_{max}>0$, we have:

$$C_{ub}(B_{max},p) - C_{lb}(B_{max},p) \le 2.58 \text{ bits}$$

Regime II: $B_{max} > E$

$$\frac{1}{2}\log\left(1+pE\right) - 2.58 \le C \le \frac{1}{2}\log\left(1+pE\right)$$

References

1 Yishun Dong, Ayfer Özgür, "Approximate Capacity of Energy Harvesting Communication with Finite Battery", submitted to *International Symposium on Information Theory*, Honolulu, 2014.
2 Yishun Dong, Farzan Farnia, Ayfer Özgür, "Near Optimal Rate Control and Approximate Capacity of Energy Harvesting Communication", in preparation for submission to *IEEE Journal on Selected Areas in Communications*, March 2014.