ON FEEDBACK IN GAUSSIAN NETWORKS

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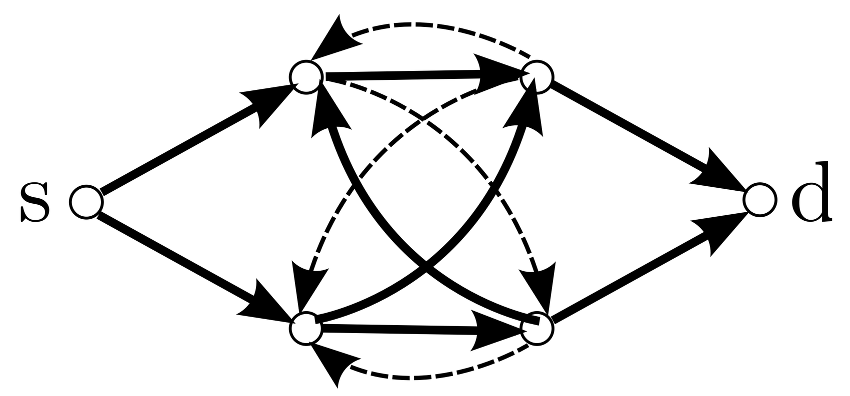
INTRODUCTION

- Feedback has been studied extensively for single-hop communication channels
- There are a lot of opportunities for "**feeding back**" information in wireless networks
- The nature of these feedback links is significantly different from the feedback models considered in the single-hop settings
- We aim to understand the role of feedback in general Gaussian networks as a model for wireless networks

MODEL



- We consider a bidirected Gaussian relay network G consisting of a set of nodes V and communication links E.
- We use multiple-input multiple-output channel model, i.e. if we let $X_v \in \mathbb{C}^{M_v}$ denote the signal transmitted by node $v \in V$ with M_v transmit antennas and let $Y_v \in \mathbb{C}^{N_v}$ denote the signal received by node $v \in V$ with N_v receive antennas. We have

$$Y_v = \sum_{u \in V} H_{vu} X_u + Z_v,$$

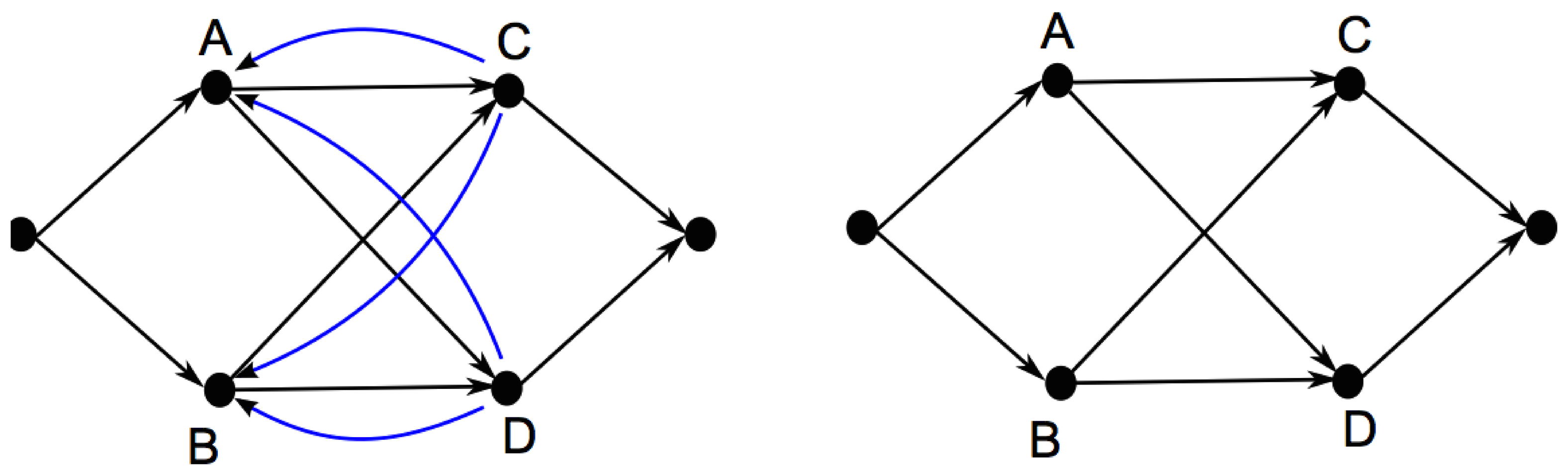
- The noise Z_v are independent and circularly symmetric Gaussian random vectors $\mathcal{N}(0, I)$.
- All nodes are subject to an average power constraint P.

TRAFFIC SCENARIOS

We consider the following traffic scenarios over the network:

- Unicast: Source nodes $s \in V$ wants to communicate to the destination node $d \in V$.
- Multiple-Access: Source nodes $s_1, s_2, \ldots, s_n \in V$ want to communicate independent messages to a destination node $d \in V$.
- Broadcast: Source node $s \in V$ wants to communicate independent messages to destination nodes $d_1, \ldots, d_n \in V$.
- Multicast: Source node $s \in V$ wants to communicate the same message to destination nodes $d_1, \ldots, d_n \in V$.
- Multiple-Unicast: Source node $s_i \in V$ wants to communicate to its destination node $d_i \in V$ for $i = 1, \ldots, n$.

UNICAST



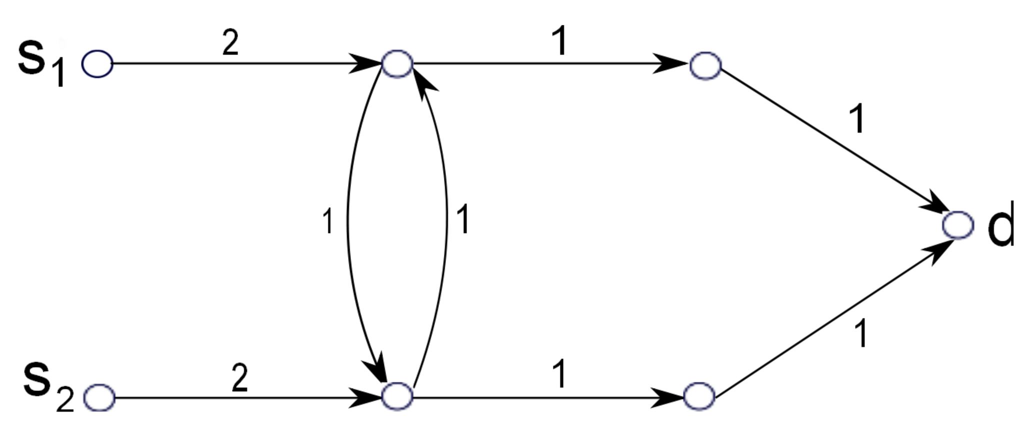
The capacity of the network G, denoted by $C(G)$, is the largest rate at which we can reliably communicate from s to d.

Theorem 1: In any Gaussian network G with capacity $C(G)$, we can identify a directed acyclic subnetwork $\tilde{G}$ whose capacity $C(\tilde{G})$ in bits/s/Hz is bounded by

$$C(G) - g \le C(\tilde{G}) \le C(G) + g$$

where $\quad g = 2\sum_{v\in V} M_v + (2 + \log 2)\sum_{v\in V} N_v$

MULTIPLE ACCESS



The capacity region $C(G)$ is the closure of jointly achievable rate pairs $R_1, \ldots, R_n$ where R_i is the communication rate from s_i to d.

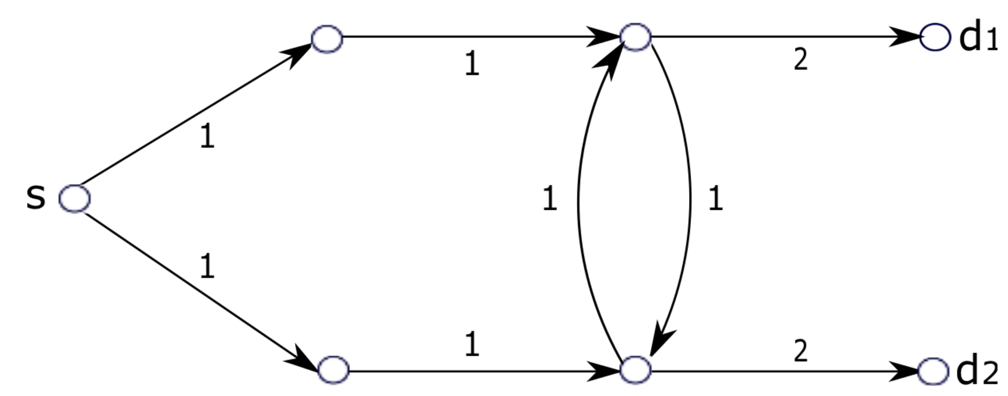
Theorem 2: Let $C(G)$ be the capacity region of a Gaussian network G with multiple access traffic. If $(R_1, R_2, \ldots, R_n) \in C(G)$, then there exists an acyclic subnetwork $\tilde{G}$ such that

$$(R_1 - g_1, R_2 - g_1, \ldots, R_n - g_1) \in C(\tilde{G})$$

where $\quad g_1 = 0.63 \sum_{v\in V}(M_v + N_v)$

However, we cannot conclude that there exists an acyclic subnetwork with the same capacity region as the original network. As a counterexample, see the above figure.

BROADCAST



The capacity region $C(G)$ is the closure of jointly achievable rate pairs $R_1, \ldots, R_n$ where R_i is the communication rate from s to d_i.

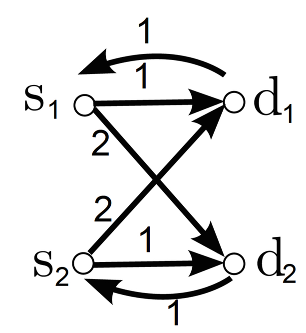
Theorem 3: Let $C(G)$ be the capacity region of a Gaussian network G with broadcast traffic. If $(R_1, R_2, \ldots, R_n) \in C(G)$, then there exists an acyclic subnetwork $\tilde{G}$ such that

$$(R_1 - g_2, R_2 - g_2, \ldots, R_n - g_2) \in C(\tilde{G})$$

where $\quad g_2 = O(M \log M), M = \sum_{v\in V} M_v + \sum_{v\in V} N_v$

However, as in the case of multiple access, the above theorem does not imply the existence of a single acyclic subnetwork whose capacity region is as large as the original network. For a counter example consider the network in the above figure.

MULTICAST AND MULTIPLE UNICAST



As a counterexample for multicast, consider the network shown for broadcast.

For multiple unicast, the classical Gaussian interference channel with feedback (the above figure) readily provides a counterexample.

REFERENCES

1 Farzan Farnia, Ayfer Özgür, "On feedback in Gaussian multi-hop networks", IEEE Information Theory and Applications Workshop 2014, San Diego

2 Bobbie Chern, Ayfer Özgür, "On information flow and feedback in relay networks", IEEE Information Theory Workshop 2013, Seville