ON FEEDBACK IN GAUSSIAN NETWORKS

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INTRODUCTION

- Feedback has been studied extensively for single-hop communication channels
- There are a lot of opportunities for "feeding back" information in wireless networks
- The nature of these feedback links is significantly different from the feedback models considered in the single-hop settings
- We aim to understand the role of feedback in general Gaussian networks as a model for wireless networks

MODEL

- We consider a bidirected Gaussian relay network $G$ consisting of a set of nodes $V$ and communication links $E$.
- We use multiple-input multiple-output channel model, i.e. if we let $X_e \in C^{M_e}$ denote the signal transmitted by node $e \in V$ with $M_e$ transmit antennas and let $Y_v \in C^{N_v}$ denote the signal received by node $v \in V$ with $N_v$ receive antennas. We have
  $$Y_v = \sum_{u \in V} H_{uv} X_u + Z_v,$$
- The noise $Z_v$ are independent and circularly symmetric Gaussian random vectors $N(0, I)$.
- All nodes are subject to an average power constraint $P$.

TRAFFIC SCENARIOS

We consider the following traffic scenarios over the network:
- Unicast: Source node $s \in V$ wants to communicate to the destination node $d \in V$.
- Multiple-Access: Source nodes $s_1, s_2, \ldots, s_n \in V$ want to communicate independent messages to a destination node $d \in V$.
- Broadcast: Source node $s \in V$ wants to communicate independent messages to destination nodes $d_1, \ldots, d_m \in V$.
- Multicast: Source node $s \in V$ wants to communicate to the same message to destination nodes $d_1, \ldots, d_n \in V$.
- Multiple-Unicast: Source node $s_i \in V$ wants to communicate to its destination node $d_i \in V$ for $i = 1, \ldots, n$.

UNICAST

The capacity of the network $G$, denoted by $C(G)$, is the largest rate at which we can reliably communicate from $s$ to $d$.

Theorem 1: In any Gaussian network $G$ with capacity $C(G)$, we can identify a directed acyclic subnetwork $\tilde{G}$ whose capacity $C(\tilde{G})$ in bits/s/Hz is bounded by

$$C(\tilde{G}) - g \leq C(G) \leq C(\tilde{G}) + g$$

where $g = 2 \sum_{v \in V} M_v + (2 + \log 2) \sum_{v \in V} N_v$.

MULTIPLE ACCESS

The capacity region $C(G)$ is the closure of jointly achievable rate pairs $R_1, \ldots, R_n$, where $R_i$ is the communication rate from $s_i$ to $d_i$.

Theorem 2: Let $C(G)$ be the capacity region of a Gaussian network $G$ with multiple access traffic. If $(R_1, R_2, \ldots, R_n) \in C(G)$, then there exists an acyclic subnetwork $\tilde{G}$ such that

$$(R_1 - g_1, R_2 - g_1, \ldots, R_n - g_1) \in C(\tilde{G})$$

where $g_1 = 0.63 \sum_{v \in V} (M_v + N_v)$.

However, we cannot conclude that there exists an acyclic subnetwork with the same capacity region as the original network. As a counterexample, see the above figure.

MULTICAST AND MULTIPLE UNICAST

As a counterexample for multicast, consider the network shown for broadcast.

For multiple unicast, the classical Gaussian interference channel with feedback (the above figure) readily provides a counterexample.

REFERENCES